ON A FUNCTION ON ELEMENT ORDERS IN FINITE GROUPS

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Let G be a periodic group. The problem of obtaining information about the structure of G by looking at the orders of its elements has been considered by many authors, from many different points of view.

In this talk we consider a finite group G, and we study the function on the element orders of G defined by

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element x.

In 2009 H. Amiri, S.M. Jafarian Amiri and M. Isaacs proved that if G has order n and C_n denotes the cyclic group of order n, then

$$\psi(G) \leq \psi(C_n),$$

and

$$\psi(G) = \psi(C_n) \quad if\ and\ only\ if \quad G \simeq C_n.$$

Other results have been obtained by H. Amiri, S.M. Jafarian Amiri, M. Amiri, Y. Marefat, A. Iranmanesh, A. Tehranian, R. Shen, G. Chen and C. Wu.

I will discuss some new results concerning the function ψ, jointly obtained with Marcel Herzog and Mercede Maj. In particular I will present some better upper bounds for $\psi(G)$ when G is not cyclic.

Some other functions on the orders of the elements of a finite group G have been recently investigated by M. Garonzi and M. Patassini.

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