Let $G$ be a periodic group. The problem of obtaining information about the structure of $G$ by looking at the orders of its elements has been considered by many authors, from many different points of view.

In this talk we consider a finite group $G$, and we study the function on the element orders of $G$ defined by

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element $x$.

In 2009 H. Amiri, S.M. Jafarian Amiri and M. Isaacs proved that if $G$ has order $n$ and $C_n$ denotes the cyclic group of order $n$, then

$$\psi(G) \leq \psi(C_n),$$

and

$$\psi(G) = \psi(C_n) \quad \text{if and only if} \quad G \cong C_n.$$

Other results have been obtained by H. Amiri, S.M. Jafarian Amiri, M. Amiri, Y. Marefat, A. Iranmanesh, A. Tehranian, R. Shen, G. Chen and C. Wu.

I will discuss some new results concerning the function $\psi$, jointly obtained with Marcel Herzog and Mercede Maj. In particular I will present some better upper bounds for $\psi(G)$ when $G$ is not cyclic.

Some other functions on the orders of the elements of a finite group $G$ have been recently investigated by M. Garonzi and M. Patassini.

References


