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On fixed point free projective groups

Dedicated to the Memory of Professor Karl STRAMBACH

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The motivation of this talk is related with the classification of the homogeneous simply connected maximal 3-geometries (so-called Thurston geometries, $\mathbf{E^3}$, $\mathbf{S^3}$, $\mathbf{H^3}$, $\mathbf{S^2\times R}$, $\mathbf{H^2\times R}$, $\mathbf{\sim SL_2R}$, **Nil**, **Sol**). The author found in [1] (see also the more popular [2], [3] with co-author colleagues, also some more details as well) a unified projective interpretation for them – in the sense of Felix Klein's Erlangen Program: Namely, each **S** of the above space geometries and its isometry group IsomS can be considered as a subspace of the projective 3-sphere: S ▪ $\mathbf{PS^3}$, where a special maximal group G = IsomS ⊠ CollPS^3 of collineations acts, leaving the above subspace S invariant.

Vice-versa, we can start with the projective geometry, namely with the classification of CollPS^3 through linear transforms of dual pairs of real 4-vector spaces ($\mathbf{V^4}$, $\boldsymbol{V_4}$, **R**, ~) = PS^3 (up to positive real multiplicative equivalence ~) by the Jordan normal forms.

First we look for a 3-parameter space of translations as fixed point groups, simply transitive on a 3-subspace. For instance, think of the classical Euclidean geometry $\mathbf{E^3}$ where the 4×4 matrix group describes the translation that carries the origin (1 0 0 0) into (*1 x y z*) in the Cartesian homogeneous coordinate system E_0, E_1, E_2, E_3. This group is simply transitive on points and fixed point free on the affine 3-subspace $A^3 = P^3 \setminus e^0$, where P^3 is obtained from PS^3 by the identification of opposite points, as usual, e^0 is the ideal plane described by the linear form $\boldsymbol{e^0}$ from the dual basis ($\boldsymbol{e^0}$, $\boldsymbol{e^1}$, $\boldsymbol{e^2}$, $\boldsymbol{e^3}$) in $\boldsymbol{V_4}$ to the vector basis ($\mathbf{e}_0$, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) in $\mathbf{V^4}$, i.e. $\mathbf{e}_i \boldsymbol{e^j} = \delta_i^j$ (the *Kronecker symbol*). That means, we get the Cartesian coordinate tetrahedron (simplex), where $\mathbf{e}_0$ describes the origin E_0, incident to the side planes e^1, e^2, e^3 by the corresponding forms. The ideal plane e^0 is opposite to the origin, and contains the ideal points E_1, E_2, E_3 of the *x, y, z* axes, respectively. Then we extend this translation group with all projective collineations leaving invariant a projective polarity (or scalar product) of signature (+, +, +, 0), all with linear transforms, as usual. Now think of the "optimal" generalizations of this, as made in the papers [1] – [6] for the above Thurston geometries.

Our intention is to investigate the other possible projective transforms, not considered yet earlier, for 3-parameter transitive translations, the possible invariant projective polarities, the possible invariant Riemann metrics, etc. We hope that we get further geometries, or interesting problems at least, in this manner.

References:

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