Regular subgroups, nilpotent algebras and congruent matrices

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In this talk we consider subgroups R of the affine group $\mathrm{AGL}_n(\mathbb{F})$, for any field $\mathbb{F}$, acting regularly on the set $\{(1, v) : v \in \mathbb{F}^n\}$ of affine points. We write the elements of such subgroups R as matrices

$$\begin{pmatrix} 1 & v \\ 0 & I_n + \delta_R(v) \end{pmatrix},$$

where $\delta_R : \mathbb{F}^n \to \mathrm{Mat}_n(\mathbb{F})$. We are interested in classifying, up to conjugation, the regular subgroups R for which the function δ_R is $\mathbb{F}$-linear. Notice that if R is abelian, then δ_R is linear.

One of the main motivations for studying this problem is because it is equivalent to the problem of classifying, up to isomorphisms, the nilpotent associative algebras of dimension n over the field $\mathbb{F}$. In fact, using tools developed for regular subgroups in [2, 4], we classified the isomorphism classes of nilpotent associative algebras of dimension $n \leq 4$ (see [3, 4]), extending the results of [1, 5] also to the nonabelian case.

Furthermore, we show how to associate to square matrices in $\mathrm{Mat}_n(\mathbb{F})$ particular regular subgroups of $\mathrm{AGL}_{n+1}(\mathbb{F})$ is such a way that conjugate subgroups correspond to projectively congruent matrices.

References

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