On CT and CSA groups and related ideas

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Abstract: A group G is commutative transitive or CT if commuting is transitive on non-trivial elements. G is conjugately separated abelian or CSA if maximal abelian subgroups are malnormal. These concepts have played a prominent role in the studies of fully residually free groups, limit groups and discriminating groups. They also play a role in the solution of the Tarski problems. CSA always implies CT however the class of CSA groups is a proper subclass of the class of CT groups. For limit groups, finitely generated elementary free groups and finitely generated torsionfree hyperbolic groups they are equivalent.

Here we examine the relationship between the two concepts. In particular, a finite CSA group must be abelian. If G is CT, then G is not CSA if and only if G contains a nonabelian subgroup H which contains a nontrivial abelian subgroup N that is normal in H. For K a field the group $PSL(2, K)$ is never CSA but is CT if $char(K) = 2$ and for fields of characteristic 0 where -1 is not a sum of two squares in K. For characteristic p, where p is an odd prime number, $PSL(2, K)$ is never CT. Infinite CT groups G with a composition series and having a nontrivial normal abelian subgroup must be monolithic with monolith a simple nonabelian CT group. Further if a group G is monolithic with monolith N isomorphic to $PSL(2, K)$ for a field of characteristic 2 and G is CT, then G is isomorphic to N.