

Syllabus for Calculus 1 (software engineering students).

The set of real numbers.

Algebraic properties. Bounded sets, upper bound, lower bound, least upper bound, greatest lower bound.

Complex numbers.

Algebraic form of complex numbers, conjugate and absolute value, operations with complex numbers in algebraic form, properties of operations. Trigonometric form of complex numbers, operations in trigonometric form. Multiplication of complex numbers in trigonometric form. The n th power and n th root.

Sequences.

Convergence and divergence of sequences, definition of $\lim_{n \rightarrow \infty} a_n = L$, finding $N(\varepsilon)$, operations on limits. Sequences diverging to plus or minus infinity. Comparison of orders of magnitude $(n^n, n!, 2^n, n^k, \log n)$. Theorems for limits. Limit of the sequences $(\sqrt[p]{p})$ where $p > 0$ and $(\sqrt[n]{n})$. Subsequences, sandwich theorem. Bounded and monotonic sequence is convergent. Recursively given sequences. Limit of the sequence: $\left(\left(1 + \frac{x}{n} \right)^n \right)$. Accumulation point, limit superior, limit inferior.

Cauchy sequences, Bolzano-Weierstrass theorem.

Functions of one variable.

Properties of functions.

Domain, range, invertibility, graph of functions, even function, odd function, periodic function. Asymptotes, monotonic function, extreme values, tangent line, convexity, inflection point.

Limit of f(x).

Definition of $\lim_{x \rightarrow a} f(x) = L$, where a and L may be finite numbers or $\pm\infty$. Right-hand and left-hand limits.

Theorems for limits: 1. Operations on limits. 2. Sandwich theorem. 3. If limit of one function is 0 and the other function is bounded, then limit of product is 0. 4. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

Continuity of f(x).

Definition of continuity of $f(x)$ at $x=a$. Continuity from left and from right. Classification of points of discontinuity. Theorems for continuous functions on a finite and closed interval and applications of these theorems (Bolzano's and Weierstrass's theorems).

Elementary functions: polynomials, rational functions, inverse functions, exponential and logarithmic functions, trigonometric and inverse trigonometric functions, hyperbolic and inverse hyperbolic functions.

Differentiability of f(x).

Definition of the derivative of f at $x=a$. Derivative function. Left and right derivative. **P**: Necessary condition of differentiability. Implicit differentiation. Rules of differentiation (derivative of the sum, difference, product and ratio). Derivative of the elementary functions. Derivative of the inverse. Higher order derivatives.

Applications of derivatives. Rolle-theorem. Lagrange's mean value theorem. L'Hopital's rule. Tangent line. The relations of derivatives with extreme values, monotonicity. Convex, concave property, inflection point. Relations between extreme values, inflection point and higher order derivatives. Sketching graphs.

Integration of $f(x)$.

Definition of antiderivative, indefinite integral. Riemann sum, partition and its norm. The definition of the definite integral. Definite integral as the limit of Riemann sum. The relation between the area and definite integral. Properties of definite integral. First and second fundamental theorems of calculus (Newton-Leibniz formula). The continuity and differentiability of the integral function.

Integration techniques. Integration by parts, integration of rational functions, integral by substitution.