Calculus 1 - Exercises 2

Limits of number sequences, application of the definitions

1. Prove the following limits using the definition, that is, provide a threshold index $N(\varepsilon)$ for every $\varepsilon > 0$.

a) $a_n = \dfrac{7n+4}{2n-1} \longrightarrow \dfrac{7}{2}$

b) $a_n = 1 + (-1)^{n+1} \cdot 2^{3-n} \longrightarrow 1$

c) $a_n = \dfrac{6n^2-2}{3n^2+3} \longrightarrow 2$

d) $a_n = \dfrac{8n^3-5}{2n^3+7} \longrightarrow 4$

e) $a_n = \dfrac{6n^2-2n}{3n^2+3} \longrightarrow 2$

f) $a_n = \dfrac{8n^3-5n^2}{2n^3+7n} \longrightarrow 4$

2. Prove the following limits using the definition, that is, provide a threshold index $N(P)$ for every $P > 0$.

a) $a_n = \dfrac{n^3+2n^2+n+2}{n^2+1} \longrightarrow \infty$

b) $a_n = \dfrac{n^3-16n}{n-4} \longrightarrow \infty$

c) $a_n = n^3 + 1000n \longrightarrow \infty$

d) $b_n = n^3 - 1000n \longrightarrow \infty$

e) $a_n = \dfrac{n^3-16n}{4-n} \longrightarrow \infty$

f) $a_n = \sqrt{n^2-n} \longrightarrow \infty$

3. Prove the following limits using the definition, that is, provide a threshold index $N(\varepsilon)$ for every $\varepsilon > 0$.

a) $a_n = \dfrac{3n^2+4n+7}{n^2+n+1} \to 3$

b) $a_n = \dfrac{n^2-10^8}{5n^6+2n^3-1} \longrightarrow 0$

c) $a_n = \dfrac{2n^3-5n}{n^3+8} \longrightarrow 2$

d) $a_n = \dfrac{2n^2+3n+6}{3n^2-1} \to \dfrac{2}{3}$

4. Formulate the following statements without negation:

a) $\lim\limits_{n\to\infty} a_n \neq A \in \mathbb{R}$

b) (a_n) is divergent