

## Calculus 1 - Exercises 3

1. Calculate the limit of the following sequences:

$$\text{a) } a_n = \frac{n+3}{4n^2+7n+6}$$

$$\text{b) } a_n = \frac{n-5n^4}{n^4+8n^3+1}$$

$$\text{c) } a_n = \frac{1-n^3}{70-n^2+n}$$

$$\text{d) } a_n = \frac{-n^7+n^6-3}{n^5-n^2+2}$$

$$\text{e) } a_n = \frac{(2n^3+3)^2}{(3n+6)^6}$$

$$\text{f) } a_n = \frac{(n+1)!}{(3-2n)n!}$$

$$\text{g) } a_n = \sqrt[3]{\frac{2n^2+6}{3n^2+2n}}$$

$$\text{h) } a_n = \frac{n^{3/2}+n^2+1}{\sqrt{1+n^2}+2\sqrt{n^3+2}}$$

$$\text{i) } a_n = \frac{\sqrt[4]{n^3+6}}{\sqrt[3]{n^5+3n+2}}$$

2. Decide whether the following sequences converge and if so, find their limit:

$$\text{a) } a_n = \sqrt{n^2+n+1} - \sqrt{n^2-n+1}$$

$$\text{b) } a_n = \sqrt{n^2-7n+1} - \sqrt{n^2-n+4}$$

$$\text{c) } a_n = \sqrt{2n^2+3n+1} - \sqrt{n^2+1}$$

$$\text{d) } a_n = (3n+1)(n - \sqrt{n^2+1})$$

$$\text{e) } a_n = \frac{1}{n - \sqrt{n^2+3n+5}}$$

$$\text{f) } a_n = \sqrt[3]{n^3+3n^2-1} - \sqrt[3]{n^3-2n^2+3n+2}$$

3. Calculate the limit of the following sequences:

$$\text{a) } a_n = \frac{\sin(n)}{n}$$

$$\text{b) } a_n = \frac{n^2-5}{2n^3+6n} \cos(n^4+5n+8)$$

$$\text{c) } a_n = \frac{\log(n+1)}{n}$$

$$\text{d) } a_n = \frac{\log_{10}(n^2)+3}{\log_3(n)}$$

$$\text{e) } a_n = \frac{(-3)^{n+1}+2^{2n+3}}{8+5^n}$$

$$\text{f) } a_n = \frac{3^{2n}+n^2+1}{3^n+9^n}$$

$$\text{g) } a_n = \frac{7^n+n^7+7}{4^n+3n^2+5}$$

$$\text{h) } a_n = \frac{4^{n-1}+n^5 \cdot 3^{n+2}}{2^{2n+3}+2^{n-2}}$$

$$\text{i) } a_n = \frac{n^3 2^n + 3^n}{2^{2n} - 3n^2}$$

$$\text{j) } a_n = \frac{2n! + n^{20}}{n^n}$$

$$\text{k) } a_n = \frac{(2^n + 7^n)^2}{n!}$$

4. True or false?

$$\text{a) If } a_n \rightarrow A \text{ then } a_n^2 \rightarrow A^2.$$

$$\text{b) If } a_n^2 \rightarrow A^2 \text{ then } a_n \rightarrow A.$$

$$\text{c) If } a_n > 0 \text{ and } b_n \rightarrow \infty \text{ then } a_n b_n \rightarrow \infty.$$

$$\text{d) If } a_n \rightarrow 0 \text{ then } \frac{1}{a_n} \rightarrow \infty.$$

$$\text{e) If } a_n \rightarrow \infty \text{ then } \frac{1}{a_n} \rightarrow 0.$$

$$\text{f) If } a_n > 0 \text{ and } (a_n) \text{ is convergent then } \lim_{n \rightarrow \infty} a_n > 0.$$

## Results

1. a) 0    b) -5    c)  $\infty$     d)  $-\infty$     e)  $\frac{4}{729}$     f)  $-\frac{1}{2}$     g)  $\left(\frac{2}{3}\right)^{1/3}$     h)  $\infty$     i) 0

2. a) 1    b) -3    c)  $\infty$     d)  $-\frac{3}{2}$     e)  $-\frac{2}{3}$     f)  $\frac{5}{3}$

3. a) 0    b) 0    c) 0    d)  $\frac{\ln 9}{\ln 10}$     e) 0    f) 1

g)  $\infty$     h)  $\frac{1}{32}$     i) 0    j) 0    k) 0