Calculus 1 - Exercises 5

Limits of real functions

1. Using the appropriate definition, prove the following limits:

a) $\lim_{x\to 1}(3x+4)=7$ b) $\lim_{x\to 2}\left(\frac{x^2-4}{x-2}+3x-1\right)=9$

c) $\lim_{x\to -2}\frac{8-2x^2}{x+2}=8$ d)* $\lim_{x\to -3}\sqrt{1-5x}=4$

2. Using the appropriate definition, prove the following limits:

$\lim_{x\to\pm\infty}\frac{1-2x}{x+3}=-2$

3. Calculate the following limits:

a) $\lim_{x\to -\infty}\left(\sqrt{x^2+x}-\sqrt{x^2+5x+3}\right)$

b) $f(x)=\frac{x^2+3x-10}{(x^2-4)^2}$, $\lim_{x\to -2} f(x)=?$ $\lim_{x\to 2} f(x)=?$

c)* $\lim_{x\to 1}\frac{x^2-1}{\sqrt{x}-\sqrt{2-x}}$ d) $\lim_{x\to\pm\infty}\frac{2x^5-3x^2+1}{x^7+4x^3+5}$

e) $\lim_{x\to 1}\frac{x-1}{\sqrt{3x^2+1}-2x}$ f) $\lim_{x\to -\infty} x\left(\sqrt{x^2+1}-\sqrt{x^2-3}\right)$

4. Calculate the following limits:

a) $\lim_{x\to 3\pm 0} 2+5\{x\}$ b) $\lim_{x\to 3\pm 0}[x-1]$

5. Prove that $\lim_{x\to 0}\cos\frac{1}{x}$ does not exist.