Calculus 1 - Exercises 6

Types of discontinuities

Where and what type of discontinuity do the following functions have?

1. $f(x) = \dfrac{(x+3)\left(x^2-2x+1\right)}{x^3+3x^2}$ 2. $f(x) = \dfrac{x^4-3x^3}{\left|\,2x^2-6x\,\right|}$

3. $f(x) = \begin{cases} \dfrac{1}{|\,4-x\,|}+\dfrac{1}{4-x}, & \text{ha } x \geq 2 \\ \dfrac{x^2-10x}{x^2-11x+10}, & \text{ha } x < 2 \end{cases}$

4. a) $f(x) = e^{-\frac{1}{x^2}}$ b) $g(x) = \dfrac{1}{1-e^x}$ c) $h(x) = \dfrac{1}{1-e^{\frac{1}{x}}}$

The limit $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$

1. Calculate the following limits:

a) $\lim\limits_{x\to 0}\dfrac{\sin(2x)}{x}$ b) $\lim\limits_{x\to 0}\dfrac{\mathrm{tg}(5x)}{x}$ c) $\lim\limits_{x\to 0}\dfrac{\mathrm{tg}(4x)}{\sin(3x)}$ d) $\lim\limits_{x\to 0}\dfrac{\sin\left(5x^2\right)}{\mathrm{tg}\left(3x^2\right)}$

e) $\lim\limits_{x\to 0}\dfrac{5x-\sin(8x)}{7x+\sin(2x)}$ f) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$ g) $\lim\limits_{x\to 0}\dfrac{\cos(5x)-1}{x\sin(7x)}$ h) $\lim\limits_{x\to 0}\dfrac{1-\cos\left(9x^2\right)}{x^2}$

i)* $\lim\limits_{x\to 0}\dfrac{\mathrm{tg}\,x-\sin x}{x^3}$ j)* $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\pi}{2}-x\right)\mathrm{tg}\,x$

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2. Where and what type of discontinuity do the following functions have?

a) $f(x) = \dfrac{\sin\,|\,x-2\,|}{x-2}$ b) $f(x) = \dfrac{\sin\left(x^2-9\right)}{x^2-2x-3}$ c) $f(x) = \dfrac{\sin\left(x^2+x-2\right)}{x^2-4}$