Calculus 1 - Exercises 7

The derivative

1. Using the definition, calculate the derivatives of the following functions at x_0.

a) $f(x) = \sqrt{6x+1}$, $x_0 = 4$ b) $f(x) = \dfrac{1}{\sqrt{2x+7}}$, $x_0 = 1$

c) $f(x) = \dfrac{1}{3x+1}$, $x_0 = -1$ d) $f(x) = \dfrac{1}{x-5}$, $x_0 = 6$

2.* Prove that $(\cos x)' = -\sin x$, $x \in \mathbb{R}$

3. Find the equation of the tangent line at the point x_0 for the following functions.

a) $f(x) = x^3 + 4$, $x_0 = 1$ b) $f(x) = 3x^2 - x$, $x_0 = 2$

4. Find the parameters a and b such that the following functions are differentiable for all real numbers x.

a) $f(x) = \begin{cases} \dfrac{1}{3x-1}, & \text{ha } x \geq 1 \\ ax + b, & \text{ha } x < 1 \end{cases}$ b) $f(x) = \begin{cases} \dfrac{a}{x^2+1}, & \text{ha } x \geq 1 \\ bx^4 + 1, & \text{ha } x < 1 \end{cases}$

5. Calculate the derivatives of the following functions.

a) $f(x) = x^7 + \dfrac{1}{x^{111}}$ b) $f(x) = x^{-7} \cdot \sqrt[5]{x}$ c) $f(x) = (1+x^2)e^x$ d) $f(x) = x^3 \sin x$

e) $f(x) = x \sin x \cos x$ f) $f(x) = \operatorname{tg} x$ g) $f(x) = \operatorname{ctg} x$ h) $f(x) = \dfrac{x^2 - 3x}{x^4 + 1}$

i) $f(x) = \dfrac{\cos x}{x^3}$ j) $f(x) = \dfrac{\sin x}{x + \cos x}$ k) $f(x) = \dfrac{x^2 + 2x - 1}{x^7 + 2x + 1}$ l) $f(x) = \operatorname{sgn} x$

6. Calculate the derivatives of the following functions.

a) $f(x) = (1+x^2)^4$ b) $f(x) = (x^3 - 3x + 8)^7$ c) $f(x) = \sqrt{1+x^6}$ d) $f(x) = \dfrac{x^2+1}{\sqrt{1+2x^2}}$

e) $f(x) = \sin(x^2)$ f) $f(x) = (\sin x)^3$ g) $f(x) = \sin^3(x^2)$ h) $f(x) = \cos^2(2x+3)$

i) $f(x) = x e^{3x} - e^{-x^2}$ j) $f(x) = \sqrt{1+e^{3x}}$ k) $f(x) = (\cos^3(x) + 3)^5$ l) $f(x) = 2^x$

m) $f(x) = \ln x + \log_3 x$ n) $f(x) = \ln(x^2+1)$ o) $f(x) = x^x$ p) $f(x) = (\sin x)^{\cos x}$

7. Let $f(x) = \sqrt[3]{x}$. Prove that $f'(0)$ does not exist.

8. Let $f(x) = \sqrt[3]{x} \cdot \sin\left(\sqrt[3]{x^2}\right)$. Calculate $f'(x)$. At $x = 0$ use the definition.

9.* Let $f(x) = |x-1| \cdot \sin(2x-2)$. Calculate $f'(x)$.

Additional exercises:

https://math.bme.hu/~nagyi/calc1/calc1-differentiation.pdf