

# Calculus 1 - Exercises 8

## Elementary functions and their inverses

1. Calculate the following limits:

a)  $\lim_{x \rightarrow 0} \frac{\arctan x}{x} = ?$

b)  $\lim_{x \rightarrow 3+0} \arctan \frac{1}{3-x} = ?$   $\lim_{x \rightarrow 3-0} \arctan \frac{1}{3-x} = ?$   $\lim_{x \rightarrow \infty} \arctan \frac{1}{3-x} = ?$

c)  $\lim_{x \rightarrow 0} x \arctan \frac{1}{x} = ?$

d)  $\lim_{x \rightarrow 0} \arctan \frac{x^2 - 3x}{3x - 9} = ?$

e)  $\lim_{x \rightarrow \infty} \arctan \frac{x^2 - 1}{2x + 3} = ?$

2. Find the types of discontinuities of the following functions:

a)  $f(x) = \operatorname{arctg}\left(\frac{1}{x}\right)$

b)  $f(x) = x \cdot \operatorname{arctg}\left(\frac{1}{x}\right)$

c)  $f(x) = (x - 2) \cdot \operatorname{arctg}\left(\frac{1}{x^2 - 3x + 2}\right)$

3. (See the lecture)

Let  $f(x) = \begin{cases} x \operatorname{arctg}\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$  and  $g(x) = \begin{cases} x^2 \operatorname{arctg}\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ b, & \text{if } x = 0 \end{cases}$

a) Find the values of the parameters  $a$  and  $b$  such that  $f$  and  $g$  are continuous at  $x = 0$ .

b) Using the definition, find  $f'(0)$  and  $g'(0)$ .

5. Calculate the derivatives of the following functions:

a)  $f(x) = \arcsin(\sqrt{1 - x^2})$

b)  $f(x) = \arccos\left(\frac{1}{1 + x^2}\right)$

c)  $f(x) = \arctan\left(\frac{1}{x^2}\right)$

d)  $f(x) = \frac{\sinh(x^3)}{\cosh^3(x)}$

e)  $f(x) = \operatorname{arsinh}(6x + 5) \ln(2 + 3x)$

f)  $f(x) = \ln(x^2 + 1) \cdot \operatorname{arcosh}(\sqrt{x})$

6. Calculate the following limits:

a)  $\lim_{x \rightarrow \infty} \frac{\sinh(2x - 5)}{\cosh(1 - 2x)}$

b)  $\lim_{x \rightarrow -\infty} \frac{\sinh(2x - 5)}{\cosh(8 - 2x)}$

c)  $\lim_{x \rightarrow \infty} \tanh(2x)$

7. Calculate the derivatives of the following functions:

a)  $f(x) = (\sin x)^{\cos x}$

b)  $g(x) = (1 + x^4)^{2x}$

## L'Hospital's rule

8. Calculate the following limits:

|                                                                                              |                                                                                              |
|----------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| a) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} 2 x^3}{\operatorname{arsh} 5 x^3} = ?$ | e) $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right) = ?$               |
| b) $\lim_{x \rightarrow 0} \frac{\arcsin 3 x^2}{\operatorname{tg}^2 x} = ?$                  | f) $\lim_{x \rightarrow +0} x^{\operatorname{tg} x} = ?$                                     |
| c) $\lim_{x \rightarrow \infty} x^2 e^{-5x} = ?$                                             | g) $\lim_{x \rightarrow -\infty} \frac{e^{8x} - 2e^{-3x}}{e^{5x} + e^{-3x}} = ?$             |
| d) $\lim_{x \rightarrow +0} \sqrt{x} \ln x^7 = ?$                                            | h) $\lim_{x \rightarrow \infty} \frac{\operatorname{sh}(3x-2)}{\operatorname{ch}(3x+4)} = ?$ |

## Some solutions

Exercise 1 (chapter 4.4, page 65 of the pdf),

Exercise 8 (chapter 4.5, page 72 of the pdf):

[https://math.bme.hu/~tasnadi/merninf\\_anal\\_1/anal1\\_gyak.pdf](https://math.bme.hu/~tasnadi/merninf_anal_1/anal1_gyak.pdf)

Exercise 2:

- a) jump discontinuity at  $x = 0$
- b) removable discontinuity at  $x = 0$
- c) removable discontinuity at  $x = 2$  and jump discontinuity at  $x = 1$