

Practice exercises 12.

1. Find the following integrals:

a) $\int \frac{x^2 - 7x + 8}{x^2} dx$

b) $\int \frac{x^6}{x^2 + 1} dx$

c) $\int \sqrt{x} \cdot \sqrt[3]{x} dx$

d) $\int \tan^2 x dx$

e) $\int \frac{-2}{5x^2 + 5} dx$

f) $\int \frac{2x + 3}{x - 2} dx$

2. Find the following integrals using that $\int f(ax + b) dx = \frac{F(ax + b)}{a} + c$, where $F'(x) = f(x)$ and $a \neq 0$.

a) $\int (4x + 1)^7 dx$

b) $\int \cos(3x - 4) dx$

c) $\int \frac{e^{2x} + 1}{e^{3x}} dx$

d) $\int e^{3x} \sinh 5x dx$

e) $\int \frac{1 - e^x}{1 + e^{\frac{x}{2}}} dx$

f) $\int \sqrt{5x - 8} dx$

g) $\int \frac{1}{\sqrt[3]{1 - 2x}} dx$

h) $\int \frac{2}{9x + 1} dx$

i) $\int \frac{2}{(9x + 1)^2} dx$

j) $\int \frac{2}{9x^2 + 1} dx$

k) $\int \frac{2}{9x^2 + 3} dx$

3. Find the following integrals (complete the squares in the denominator):

a) $\int \frac{1}{x^2 + 2x + 6} dx$

b) $\int \frac{1}{2x^2 - 12x + 23} dx$

c) $\int \frac{1}{x^2 - 6x + 1} dx$

d) $\int \frac{1}{\sqrt{x^2 - 4x + 40}} dx$

e) $\int \frac{1}{\sqrt{x^2 + 6x}} dx$

4. Find the following integrals using that $\int \frac{f'(x)}{f(x)} dx = \ln | f(x) | + c$.

a) $\int \frac{5x^2}{5 - 4x^3} dx$

b) $\int \frac{x^3 + 2x}{x^4 + 4x^2 + 1} dx$

c) $\int \tan x dx$

d) $\int \frac{e^{3x}}{e^{3x} + 6} dx$

e) $\int \frac{1}{x \ln x} dx$

f) $\int \frac{1}{(1 + x^2) \arctan x} dx$

5. Find the following integrals using that $\int f'(x) f^\alpha(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha + 1} + c$.

a) $\int x^2 (4x^3 + 5)^{100} dx$

b) $\int \frac{x}{\sqrt{4 - x^2}} dx$

c) $\int \frac{2x + 1}{\sqrt[4]{x^2 + x + 3}} dx$

d) $\int \frac{9x^2}{\sqrt{1 - x^3}} dx$

e) $\int \frac{\cos x}{\sqrt{2 + \sin x}} dx$

f) $\int \sin^7 x \cos x dx$

g) $\int \frac{\cos x}{\sin^2 x} dx$

h) $\int \frac{\cosh x}{\sqrt[3]{\sinh^2 x}} dx$

i) $\int \frac{\tan x}{\cos^2 x} dx$

j) $\int \frac{\ln x}{x} dx$

k) $\int \frac{\sqrt{\ln^3 x}}{x} dx$

l) $\int \frac{e^x}{e^{2x} + 2e^x + 1} dx$

6. Find the following integrals:

a) $\int \sin^2(1 - 2x) dx$

b) $\int \sin^3 x dx$

c) $\int \cos^4 x dx$

d) $\int \cos^5 x dx$

e) $\int \sin^6 x \cos^3 x dx$

7. Find the following integrals using that $\int f(g(x)) \cdot g'(x) dx = F(g(x)) + c$, where $F'(x) = f(x)$.

a) $\int x \sin(2x^2) dx$ b) $\int 3x e^{2x^2+1} dx$ c) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$ d) $\int \frac{\ln \sqrt{x}}{x} dx$

8. Use the integration by parts method to find the following integrals.

a) $\int x e^{-x} dx$ b) $\int x^2 e^x dx$ c) $\int x^2 \cos x dx$ d) $\int x^2 \sin 2x dx$
 e) $\int x \ln x dx$ f) $\int \ln^2 x dx$ g) $\int \arctan x dx$ h) $\int x \arctan x dx$
 i) $\int \arcsin x dx$ j) $\int e^x \cos x dx$ k) $\int \sinh 3x \cos 2x dx$ l) $\int \frac{\ln \cos x}{\cos^2 x} dx$

9. Find the integrals of the following rational functions.

a) $\int \frac{x^3}{x^2+1} dx$ b) $\int \frac{1}{x^2-2x+3} dx$ c) $\int \frac{x}{x^2-2x+3} dx$
 d) $\int \frac{x}{x^2+x+1} dx$ e) $\int \frac{1}{x^2+2x} dx$ f) $\int \frac{2x+5}{x^2-4x+3} dx$
 g) $\int \frac{x^2}{(x-1)(x+1)^2} dx$ h) $\int \frac{2x^2+x+4}{x^3+x} dx$ i) $\int \frac{4x+4}{x^2(x^2+1)} dx$

10. Use the substitution $u = ax + b$ or $x = \sin t$ to find the following integrals.

a) $\int \frac{1}{4x^2+4x+2} dx$ b) $\int \frac{1}{4+(7x+3)^2} dx$ c) $\int \frac{1}{\sqrt{2x-x^2}} dx$
 d) $\int \frac{x^2}{\sqrt{1-x^2}} dx$ e) $\int \frac{4x^2}{(1-x^2)^{\frac{3}{2}}} dx$

11. Use the substitution $t = e^x$ to find the following integrals.

a) $\int \frac{6}{e^x-3} dx$ b) $\int \frac{e^x}{1-e^{2x}} dx$ c)* $\int \frac{1}{1-e^{3x}} dx$

12. Use the substitution $t = \sqrt{ax+b}$ in cases a) and b) and $t = \sqrt[4]{x}$ in case c) to find the following integrals.

a) $\int \frac{x}{\sqrt{x+1}} dx$ b) $\int \frac{\sqrt{2x+1}}{x+3} dx$ c) $\int \frac{1}{\sqrt{x} + \sqrt[4]{x}} dx$

13.* Use the substitution $t = \tan(x/2)$ to find the following integrals.

a) $\int \frac{1}{1+\sin x} dx$ a) $\int \frac{1}{1-\cos x} dx$ a) $\int \frac{\cos x}{1-\cos x} dx$ a) $\int \frac{1}{\sin x - \cos x} dx$

14.* Use the substitution $x = \sinh t$ or $x = \cosh t$ to find the following integrals.

a) $\int \sqrt{x^2-1} dx$ b) $\int \sqrt{x^2+1} dx$ c) $\int \sqrt{4x^2-2} dx$