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# Calculus 1 - Homework 1.

**1. (3 points)** Decide whether the statement is true or false and write down the negation of the statement:  $\forall r > 0 (\forall x \in \mathbb{R} (\exists q \in \mathbb{Q} (|x - q| < r)))$ .

**2. (4 points)** Let  $a_0 = 5$  and  $a_{n+1} = 8 - \frac{12}{a_n}$ . Prove that  $\forall n \in \mathbb{N} (2 \leq a_n \leq 6)$ .

**3. (4 points)** What is the maximum value of  $xy$  if  $x, y \geq 0$  and  $2x + 3y = 10$ ?

**4. (4 points)** Given a right angled triangle, its sides are  $a, b$  and  $c$  where  $c$  is the hypotenuse. Prove that  $a + b \leq \sqrt{2} \cdot c$ . When does equality hold?

**5. (4 points)** Let  $a_n = \frac{6n^4 - n^3 + 100}{2n^4 + n - 1000}$ . Find the limit of  $a_n$  and provide a threshold index  $N$  for  $\varepsilon = 0.01$ .

**6. (3 points)** Prove that if  $\lim_{n \rightarrow \infty} a_n = \infty$ , then  $\lim_{n \rightarrow \infty} \sqrt[k]{a_n} = \infty$  for all  $k \in \mathbb{N}$ .

**7. (3 points)** Find the limit of the sequence  $a_n = \sqrt{n^2 + n - 2} - \sqrt{n^2 - 2n + 3}$ .

Deadline: September 27th