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# Calculus 1 - 10

## Quiz questions - Basic topological concepts

Are the following statements true or false?

1. Let  $H \subset \mathbb{R}$ .

- a) If  $x \in H$ , then  $x$  is an interior point of  $H$ .
- b) If  $x \in H$ , then  $x$  cannot be a boundary point of  $H$ .

2. Let  $H \subset \mathbb{R}$ . If  $x \notin H$ , then  $x$  cannot be

- a) an interior point of  $H$ ;
- b) a boundary point of  $H$ ;
- c) a limit point of  $H$ ;
- d) an isolated limit point of  $H$ .

3. a) If  $x$  is an interior point of  $H \subset \mathbb{R}$ , then  $x$  is a limit point of  $H$ .

b) If  $x$  is a boundary point of  $H \subset \mathbb{R}$ , then  $x$  is a limit point of  $H$ .

4. a) If  $x$  is a limit point of  $H \subset \mathbb{R}$ , then  $x$  is a boundary point of  $H$ .

b) If  $x$  is a limit point of  $H \subset \mathbb{R}$ , then  $x$  is an interior point or a boundary point of  $H$ .

5. a) If  $x$  is an isolated point of  $H \subset \mathbb{R}$ , then  $x$  is boundary point of  $H$ .

b) If  $H \subset \mathbb{R}$ ,  $x \in H$  and  $x$  is not an isolated point of  $H$ , then  $x$  is an interior point of  $H$ .

6. a) If  $x \in \mathbb{R}$  has a neighbourhood that contains infinitely many points of  $H \subset \mathbb{R}$ , then  $x$  is a limit point of  $H$ .

b) If every neighbourhood of  $x \in \mathbb{R}$  contains infinitely many points of  $H \subset \mathbb{R}$ , then  $x$  is a limit point of  $H$ .

7. There exists a set  $H \subset \mathbb{R}$  which has

- a) no interior points;
- b) no boundary points;
- c) no limit points;
- d) no isolated points.

8. There exists a set  $H \subset \mathbb{R}$  such that all points of  $H$  are

- a) interior points;
- b) boundary points;
- c) limit points;
- d) isolated points.

9. There exists a set  $H \subset \mathbb{R}$  which has

- a) exactly one interior point;
- b) exactly one limit point;
- c) exactly one boundary point.

10. There exists a set  $H \subset \mathbb{R}$  which is equal to the

- a) set of its interior points;
- b) set of its limit points;
- c) set of its boundary points.

11. a) The set  $H = [0, 1] \cap \mathbb{Q}$  is open.  
b) The set  $H = [0, 1] \cap \mathbb{Q}$  is closed.  
c) The set  $H = \left\{ \frac{1}{n} : n \in \mathbb{N}^+ \right\}$  is closed.
12. a) If the set  $H \subset \mathbb{R}$  is open, then every point of  $H$  is an interior point.  
b) If the set  $H \subset \mathbb{R}$  is closed, then every point of  $H$  is a boundary point.
13. a) If the set  $H \subset \mathbb{R}$  is closed, then every point of  $H$  is a limit point.  
b) If every point of the set  $H \subset \mathbb{R}$  is a limit point, then  $H$  is closed.
14. a) If the set  $H \subset \mathbb{R}$  is closed, then it contains all of its limit points.  
b) If the set  $H \subset \mathbb{R}$  contains all of its limit points, then it is closed.
15. a) If the set  $H \subset \mathbb{R}$  has finitely many points, then it has no limit points.  
b) If the set  $H \subset \mathbb{R}$  has infinitely many points, then it has at least one limit point.  
c) If the set  $H \subset \mathbb{R}$  is bounded and has infinitely many points, then it has at least one limit point.