2. The Voter Process

Modeling the Voter Process

We begin with a set of sites, known as **voters**, arranged in an m by n rectangular integer lattice:

$$V = \{0, 1, \ldots, m-1\} \times \{0, 1, \ldots, n-1\}$$

Each element of V has four **neighbors**; the neighbors of (i, j) are $\{(i+1, j), (i-j, j), (i, j+1), (i, j-1)\}$, where the arithmetic operations in the first coordinate are interpreted *modulo* m:

- $(m-1)+1 = 0$
- $0-1 = m-1$

and where the arithmetic operations in the second coordinate are interpreted *modulo* n:

- $(n-1)+1 = 0$
- $0-1 = n-1$

With this neighborhood structure, our set of sites is topologically a **torus**, a doughnut-shaped surface. You can imagine constructing a torus from a rectangle by first connecting two opposite edges to make a cylinder and then connecting the circular edges of the cylinder.

Each site, at any time, must be in one of a finite set of **states** S. The elements of the state space S are interpreted as the possible positions of the voters on some issue, but they can also be conveniently thought of as colors.

Time is discrete, and the dynamics of the voter process are as follows: at each time unit,

1. A site is selected at random (each site is equally likely to be selected).
2. A neighbor of this site is selected at random (each of the 4 neighbors is equally likely to be selected).
3. The state (color) of the selected site is changed to that of the selected neighbor.

Initially, each site, independently, is given a state randomly selected among the set of states; thus we have a random, uniform initial configuration.

1. Run the 10 by 5 voter process for 100 time units, updating every time. Make sure that you understand how the process works.

Our main interest is in the asymptotic behavior of the process. In particular, will the process eventually reach consensus (all sites the same color), or can the process go on forever with more than 2 colors?

2. Run the 10 by 5 voter process 10,000 times, updating every 100 runs. Note the asymptotic behavior.

The main theoretical result is that the voter process will eventually reach consensus; that is, all sites will eventually be the same color.

3. In the voter process, select the 10 by 5 voter array and set the process to stop when a color dies. Run the simulation until all sites are the same color. Note the time that each color dies.

4. In the voter process, select the 20 by 10 voter array and set the process to stop when a color dies. Run the simulation until all sites are the same color. Note the time that each color dies.

5. In the voter process, select the 50 by 25 voter array and set the process to stop when a color dies. Run the simulation until all sites are the same color. Note the time that each color dies. (This may take a very long time).
