1. Introduction

We will consider a process in which points occur randomly in time. The phrase **points in time** is generic and could represent, for example:

- The times when a sample of radioactive material emits particles
- The times when customers arrive at a service station
- The times when file requests arrive at a server computer
- The times when accidents occur at a particular intersection
- The times when a device fails and is replaced by a new device

It turns out that under some basic assumptions that deal with independence and uniformity in time, a *single*, one-parameter probability model governs all such random processes. This is an amazing result, and because of it, the **Poisson process** (named after Simeon Poisson) is one of the most important in probability theory.

Random Variables

There are two collections of random variables that can be used to describe the process; these collections are inverses of one another in a certain sense.

First, let T_k denote the time of the k^{th} arrival for $k \in \mathbb{N}_+$. Next, let N_t denote the number of arrivals in the interval $(0, t]$ for $t \geq 0$. Note that

$$(N_t \geq k) \Leftrightarrow (T_k \leq t)$$

since each of these events means that there are at least k arrivals in the interval $(0, t]$.

The Basic Assumption

The assumption that we will make can be described intuitively (but imprecisely) as follows: If we fix a time t, whether constant or one of the arrival times, then the process *after* time t is independent of the process *before* time t and behaves probabilistically just like the original process. Thus, the random process has a **strong renewal property**. Making the strong renewal assumption precise will enable use to completely specify the probabilistic behavior of the process, up to a single, positive parameter.

1. Think about the strong renewal assumption for each of the specific applications given above.

As a first step, note that the strong renewal assumption means that the times between arrivals, known as **interarrival times**, must be independent, identically distributed random variables. Formally, the sequence of interarrival times $(X_1, X_2, \ldots)$ is defined as follows:

1. $X_1 = T_1$
2. $X_k = T_k - T_{k-1}$ for all $k \in \{2, 3, \ldots\}$

The strong renewal assumption will allow us to find the distribution and essential properties of each of the following in turn:

- the interarrival times
- the arrival times
- the counting variables

The Poisson process is the most important example of a type of random process known as a **renewal process**. For such processes generally, the renewal property must only be satisfied at the arrival times; thus, the interarrival times are independent and identically distributed. A separate chapter on Renewal Processes explores the processes in detail.
