Answers to Selected Exercises

4. Special Distributions

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1. Introduction

☑ 1.6. $f(x) = \frac{1}{b} \exp\left(-\frac{x-a}{b}\right), \; x > a$

☑ 1.7. $f(x) = \dfrac{1}{\pi b \left(1 + \frac{x-a}{b}\right)^2}, \; x \in \mathbb{R}$

2. The Normal Distribution

☑ 2.22. Let X denote the volume of beer in liters

    a. $\mathbb{P}(X > 0.48) = 0.9772$
    b. $x_{0.95} = 0.51645$

☑ 2.23. Let X denote the radius of the rod and Y the radius of the hole. $\mathbb{P}(Y - X < 0) = 0.0028$.

☑ 2.24. Let X denote the combined weight of the 5 peaches, in ounces. $\mathbb{P}(X > 45) = 0.0127$.

3. The Gamma Distribution

☑ 3.8. $\mathbb{P}(X > 3) = \frac{17}{2} e^{-3} \approx 0.432$

☑ 3.9.

   a. $Q_1 = 0.287$, $Q_2 = 0.693$, $Q_3 = 1.396$, $Q_3 - Q_1 = 1.109$,
   b. $Q_1 = 0.961$, $Q_2 = 1.678$, $Q_3 = 2.692$, $Q_3 - Q_1 = 1.731$,
   c. $Q_1 = 1.727$, $Q_2 = 2.674$, $Q_3 = 3.920$, $Q_3 - Q_1 = 2.193$,

☑ 3.14. Let X denote the petal length in centimeters.

   a. $\mathbb{E}(X) = 4$
   b. $\mathrm{sd}(X) = 2$

☑ 3.21. Let X denote the lifetime in hours.

   a. $\mathbb{P}(X > 300) = 13\, e^{-3} \approx 0.6472$
   b. $\mathbb{E}(X) = 400$
   c. $\mathrm{sd}(X) = 200$

☑ 3.26.

   a. $\mathbb{P}(18 < X < 25) = 0.3860$, $\mathbb{P}(18 < X < 25) \approx 0.4095$
   b. $y_{80} = 25.038$, $y_{80} \approx 25.325$

4. The Chi-Square Distribution

☑ 4.5.

   a. $Q_1 = 0.102$, $Q_2 = 0.455$, $Q_3 = 1.323$, $Q_3 - Q_1 = 1.221$,
   b. $Q_1 = 0.575$, $Q_2 = 1.386$, $Q_3 = 2.773$, $Q_3 - Q_1 = 2.198$,
   c. $Q_1 = 2.675$, $Q_2 = 4.351$, $Q_3 = 6.626$, $Q_3 - Q_1 = 3.951$,
   d. $Q_1 = 6.737$, $Q_2 = 9.342$, $Q_3 = 12.549$, $Q_3 - Q_1 = 5.812$,

☑ 4.14. Let Z denote the distance from the missile to the target. $\mathbb{P}(Z < 20) = 1 - e^{-2} \approx 0.8647$.

☑ 4.16.

   a. $\mathbb{P}(15 < X < 20) = 0.3252$, $\mathbb{P}(15 < X < 20) \approx 0.3221$
   b. $x_{0.75} = 21.605$, $x_{0.75} \approx 22.044$

5. The Student t Distribution

☑ 5.6.

    a. $Q_1 = -1, \ Q_2 = 0, \ Q_3 = 1, \ Q_3 - Q_1 = 2,$
    b. $Q_1 = -0.816, \ Q_2 = 0, \ Q_3 = 0.816, \ Q_3 - Q_1 = 1.632,$
    c. $Q_1 = -0.727, \ Q_2 = 0, \ Q_3 = 0.727, \ Q_3 - Q_1 = 1.454,$
    d. $Q_1 = -0.7, \ Q_2 = 0, \ Q_3 = 0.7, \ Q_3 - Q_1 = 1.4,$

☑ 5.13.

    a. $\mathbb{P}(-0.8 < T < 1.2) = 0.650, \ \mathbb{P}(-0.8 < T < 1.2) \approx 0.673$
    b. $x_{0.90} = 1.372, \ x_{0.90} \approx 1.281$

6. The F distribution

☑ 6.4.

    a. $Q_1 = 0.528, \ Q_2 = 1, \ Q_3 = 1.895, \ Q_3 - Q_1 = 1.367,$
    b. $Q_1 = 0.529, \ Q_2 = 932, \ Q_3 = 1.585, \ Q_3 - Q_1 = 1.056,$
    c. $Q_1 = 0.631, \ Q_2 = 1.073, \ Q_3 = 1.890, \ Q_3 - Q_1 = 1.259,$
    d. $Q_1 = 0.645, \ Q_2 = 1, \ Q_3 = 1.551, \ Q_3 - Q_1 = 0.906,$

8. The Beta Distribution

☑ 8.14.

    a. $Q_1 = 0.25, \ Q_2 = 0.5, \ Q_3 = 0.75, \ Q_3 - Q_1 = 0.5,$
    b. $Q_1 = 0.091, \ Q_2 = 0.206, \ Q_3 = 0.370, \ Q_3 - Q_1 = 0.279,$
    c. $Q_1 = 0.630, \ Q_2 = 0.794, \ Q_3 = 0.9009, \ Q_3 - Q_1 = 0.279,$
    d. $Q_1 = 0.194, \ Q_2 = 0.314, \ Q_3 = 0.454, \ Q_3 - Q_1 = 0.260,$
    e. $Q_1 = 0.546, \ Q_2 = 0.686, \ Q_3 = 0.806, \ Q_3 - Q_1 = 0.260,$
    f. $Q_1 = 0.379, \ Q_2 = 0.5, \ Q_3 = 0.621, \ Q_3 - Q_1 = 0.242,$

9. The Weibull Distribution

☑ 9.7. $Q_1 = 0.5364, \ Q_2 = 0.8326, \ Q_3 = 1.1774, \ Q_3 - Q_1 = 0.6411,$
☑ 9.24.

a. $\mathbb{P}(T > 1500) = 0.1966$

b. $\mathbb{E}(T) = 940.656$, $\text{sd}(T) = 787.237$

c. $h(t) = 0.0.000301\, t^{0.2}$

10. The Zeta Distribution

☑ 10.3. $\mathbb{P}(X > 4) = 1 - \frac{49}{6\pi^2} \approx 0.1725$

☑ 10.7.

a. $\mathbb{E}(X) = 1.1106$

b. $\text{sd}(X) = 0.5351$

11. The Pareto Distribution

☑ 11.6. $Q_1 = 1.1006$, $Q_2 = 1.2599$, $Q_3 = 1.5874$, $Q_3 - Q_1 = 0.4868$

☑ 11.16. Let X denote income.

a. $\mathbb{P}(2000 < X < 4000) = 0.1637$ so the proportion is 16.37%

b. $Q_2 = 1259.92$

c. $Q_1 = 1100.64$, $Q_3 = 1587.40$, $Q_3 - Q_1 = 486.76$

d. $\mathbb{E}(X) = 1500$

e. $\text{sd}(X) = 866.03$

f. $F^{-1}(0.9) = 2154.43$

12. The Logistic Distribution

☑ 12.2. $\mathbb{P}(-1 < X < 2) = 0.6119$

☑ 12.7. $Q_1 = -1.0986$, $Q_2 = 0$, $Q_3 = 1.0986$, $Q_3 - Q_1 = 2.1972$

☑ 12.8. $F^{-1}(0.1) = -2.1972$, $F^{-1}(0.9) = 2.1972$

13. The Lognormal Distribution

☑ 13.6. $\mathbb{P}(X > 20) = 0.1497$

☑ 13.7. $Q_1 = 0.5097$, $Q_2 = 1$, $Q_3 = 1.9621$, $Q_3 - Q_1 = 1.4524$

☑ 13.11.

a. $\mathbb{E}(X) = e^{\frac{5}{2}} \approx 12.1825$

b. $\text{sd}(X) = \sqrt{e^6 - e^5} \approx 15.9629$

15. Benford's Law

☑ 15.4.

a. $f(y) = \dfrac{1}{0.2303\,y}$ for $y \in \left[\frac{1}{10}, 1\right)$.

b. $\mathbb{E}(Y) = 0.3909$, $\text{var}(Y) = 0.0622$,

☑ 15.6.

a.

n	$\mathbb{P}(N_1 = n)$
1	0.3010
2	0.1761
3	0.1249
4	0.0969
5	0.0792
6	0.0669
7	0.0580
8	0.0512
9	0.0458

b. $\mathbb{E}(N_1) = 3.4402$, $\text{var}(N_1) = 6.0567$

☑ 15.8.

$$\mathbb{P}(N_1 = n_1,\ N_2 = n_2)$$

$n_2 \backslash n_1$	1	2	3	4	5	6	7	8	9
0	0.0414	0.0212	0.0142	0.0107	0.0086	0.0072	0.0062	0.0054	0.0048
1	0.0378	0.0202	0.0138	0.0105	0.0084	0.0071	0.0061	0.0053	0.0047
2	0.0348	0.0193	0.0134	0.0102	0.0083	0.0069	0.0060	0.0053	0.0047
3	0.0322	0.0185	0.0130	0.0100	0.0081	0.0068	0.0059	0.0052	0.0046
4	0.0300	0.0177	0.0126	0.0098	0.0080	0.0067	0.0058	0.0051	0.0046

5	0.0280	0.0170	0.0122	0.0092	0.0078	0.0066	0.0058	0.0051	0.0045
6	0.0263	0.0164	0.0119	0.0093	0.0077	0.0065	0.0057	0.0050	0.0045
7	0.0248	0.0158	0.0116	0.0091	0.0076	0.0064	0.0056	0.0050	0.0045
8	0.0235	0.0152	0.0113	0.0090	0.0074	0.0063	0.0055	0.0049	0.0044
9	0.0223	0.0147	0.0110	0.0088	0.0073	0.0062	0.0055	0.0049	0.0044

☑ 15.10.

a.

n	$\mathbb{P}(N_2 = n)$
0	0.1197
1	0.1139
2	0.1088
3	0.1043
4	0.1003
5	0.0967
6	0.0934
7	0.0904
8	0.0876
9	0.0850

b. $\mathbb{E}(N_2) = 4.1874$, $\operatorname{var}(N_2) = 0.8254$

☑ 15.12.

a. 0.0008171
b. 0.001377
c. 0.003205