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Pataki, Gergely (H-LAJO-IF)
On the convergence of some particular series. (English summary)
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Denoting by S the family of all sequences whose terms belong to the interval $(0,1]$, this paper presents a number of elementary theorems relating to series constructed from sequences in S. One of the first results (Theorem 1.3) takes a fixed $(x_n)\in S$ and the assertion: $(*)$ there exists $a>0$ such that $\sum a^{1/x_n}<\infty$, and proves three other assertions equivalent to $(*)$. This leads to the related result (Theorem 2.2): If $(x_n)\in S$ and if $\sum a_n^{1/x_n}<\infty$ for all $(a_n)\in S$ with $\lim a_n=0$, then $\lim x_n=0$. Counterexamples show that the converse of this theorem need not be true. Other similar types of results allow the basic sequences to belong to wider intervals, for instance (Theorem 4.2): If $b_n>0$, $\sum b_n<\infty$, and if $y_n\in\mathbb{R}$ with $\sup y_n<\infty$, then $\sum b_n^{1-y_n/n}<\infty$.

Reviewed by *D. C. Russell*