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On the convergence of some particular series. (English summary)

Denoting by $S$ the family of all sequences whose terms belong to the interval $(0, 1]$, this paper presents a number of elementary theorems relating to series constructed from sequences in $S$. One of the first results (Theorem 1.3) takes a fixed $(x_n) \in S$ and the assertion: (*) there exists $a > 0$ such that $\sum a^{1/x_n} < \infty$, and proves three other assertions equivalent to (*). This leads to the related result (Theorem 2.2): If $(x_n) \in S$ and if $\sum a^n/x_n < \infty$ for all $(a_n) \in S$ with $\lim a_n = 0$, then $\lim x_n = 0$. Counterexamples show that the converse of this theorem need not be true. Other similar types of results allow the basic sequences to belong to wider intervals, for instance (Theorem 4.2): If $b_n > 0$, $\sum b_n < \infty$, and if $y_n \in \mathbb{R}$ with $\sup y_n < \infty$, then $\sum b_n^{1-y_n/n} < \infty$.

Reviewed by D. C. Russell

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