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Pataki, Gergely; Szász, Árpád

Characterizations of nonexpansive multipliers on partially ordered sets. (English)

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A function f from a subset \mathcal{D} of a poset (partially ordered set) \mathcal{A} into \mathcal{A} is nonexpansive if $f(d) \leq d$ for all $d \in \mathcal{D}$. A function f from a subset \mathcal{D} of a poset \mathcal{A} into \mathcal{A} is a multiplier if $f(d) \wedge e = f(e) \wedge d$ for all $d, e \in \mathcal{D}$. A function f from a subset \mathcal{D} of a poset \mathcal{A} into a poset \mathcal{B} is nondecreasing if $f(d) \leq f(e)$ for all $d, e \in \mathcal{D}$ with $d \leq e$. A function f from a subset \mathcal{D} of a set \mathcal{A} into \mathcal{A} is quasi-idempotent if $f(f(d)) = f(d)$ for all $d \in \mathcal{D}$ with $f(d) \in \mathcal{D}$. A quasi-idempotent function f from a subset \mathcal{D} of a set \mathcal{A} into \mathcal{A} is idempotent if $f[\mathcal{D}] \subseteq \mathcal{D}$. A function f from a subset \mathcal{D} of a poset \mathcal{A} into \mathcal{A} is a quasi-interior operator if it is nonexpansive, nondecreasing and quasi-idempotent. A quasi-interior operator f from a subset \mathcal{D} of a poset \mathcal{A} into \mathcal{A} is an interior operator if $f[\mathcal{D}] \subseteq \mathcal{D}$. A nonempty subset \mathcal{B} of a poset \mathcal{A} is a semilattice in \mathcal{A} if $d \wedge e$ exists in \mathcal{A} and belongs to \mathcal{B} for all $d, e \in \mathcal{B}$. A function f from a subset \mathcal{D} of a poset \mathcal{A} into a poset \mathcal{B} is quasi-multiplicative if $f(d \wedge e) = f(d) \wedge f(e)$ for all $d, e \in \mathcal{D}$ such that $d \wedge e$ exists in \mathcal{A} and belongs to \mathcal{D} . A quasi-multiplicative function f from a subset \mathcal{D} of a poset \mathcal{A} into a poset \mathcal{B} is multiplicative if \mathcal{D} is a semilattice in \mathcal{A} .

The authors present basic characterizations of the defined sorts of functions and operators and establish several relationships between them, thus obtaining an extension and a supplementation of some previous results of *G. Szász* [“Die Translationen der Halbverbände”, *Acta Sci. Math.* 17, 165-169 (1956; Zbl 0078.02001); “Translationen der Verbände”, *Acta Fac. Rer. Nat. Univ. Comenianae, Math.* 5, 449-453 (1961; Zbl 0112.01901)], *G. Szász* and *J. Szendrei* [“Über die Translation der Halbverbände”, *Acta Sci. Math.* 18, 44-47 (1957; Zbl 0078.02002)], *M. Kolibiar* [“Bemerkungen über Translationen der Verbände”, *Acta Fac. Rer. Nat. Univ. Comenianae, Math.* 5, 455-458 (1961; Zbl 0113.01901)], *W. H. Cornish* [“The multiplier extension of a distributive lattice”, *J. Algebra* 32, 339-355 (1974; Zbl 0318.06016)] and *Á. Szász* [“Partial multipliers on partially ordered sets”, Technical Report, Inst. Math. Inf., Univ. Debrecen 98/8, 1-28 (1998)].

Jarmila Hedlíková (Bratislava)

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