

Pataki, Gergely

On a generalized infimal convolution of set functions. (English) Zbl 1321.03060
Ann. Math. Sil. 27, 99-106 (2013).

Summary: Having in mind the ideas of J. Moreau, T. Strömberg and Á. Szász, for any function f and g of one power set $\mathcal{P}(\mathcal{X})$ to another $\mathcal{P}(\mathcal{Y})$, we define an other function $(f * g)$ of $\mathcal{P}(\mathcal{X})$ to $\mathcal{P}(\mathcal{Y})$ such that

$$(f * g)(A) = \bigcap \{f(U) \cup g(V) : A \subset U \cup V \subset X\}$$

for all $A \subset X$. Thus $(f * g)$ is a generalized infimal convolution of f and g .

We show that if f and g preserve arbitrary unions, then $(f * g)$ also preserves arbitrary unions. Moreover, if F and G are relations on X to Y such that

$$F(x) = f(\{x\}) \quad \text{and} \quad G(x) = g(\{x\})$$

for all $x \in X$, then

$$(f * g)(A) = (F \cap G)[A]$$

for all $A \subset X$.

MSC:

03E20 Other classical set theory (logic)

06A06 Partial order

46A22 Theorems of Hahn-Banach type; extension and lifting of functionals and operators

26E25 Set-valued real functions