

## Properties of entropy and conditional entropy

- (i)  $H(\alpha \vee \gamma | \beta) = H(\alpha | \beta) + H(\gamma | \alpha \vee \beta)$ .
- (ii)  $H(\alpha \vee \gamma) = H(\alpha) + H(\gamma | \alpha)$ .
- (iii) If  $\alpha \leq \gamma$ , then  $H(\alpha | \beta) \leq H(\gamma | \beta)$ .
- (iv) If  $\alpha \leq \gamma$ , then  $H(\alpha) \leq H(\gamma)$ .
- (v) If  $\alpha \leq \gamma$ , then  $H(\beta | \alpha) \geq H(\beta | \gamma)$ .
- (vi)  $H(\alpha) \geq H(\alpha | \gamma)$ .
- (vii)  $H(\alpha \vee \gamma | \beta) \leq H(\alpha | \beta) + H(\gamma | \beta)$ .
- (viii)  $H(\alpha \vee \gamma) \leq H(\alpha) + H(\gamma)$ .
- (ix)  $H(T^{-1}\alpha | T^{-1}\beta) = H(\alpha | \beta)$ .
- (x)  $H(T^{-1}\alpha) = H(\alpha)$ .
- (xi)  $H(\alpha | \gamma) = 0 \Leftrightarrow H(\alpha \vee \gamma) = H(\gamma) \Leftrightarrow \alpha \leq \gamma$ .
- (xii)  $H(\alpha | \gamma) = H(\alpha) \Leftrightarrow H(\alpha \vee \gamma) = H(\alpha) + H(\gamma) \Leftrightarrow \alpha \perp \gamma$ .

## Properties of entropy of a transformation

- (1)  $h(T, \alpha) \leq H(\alpha)$ .
- (2)  $h(T, \alpha \vee \beta) \leq h(T, \alpha) + h(T, \beta)$ .
- (3) If  $\alpha \leq \beta$ , then  $h(T, \alpha) \leq h(T, \beta)$ .
- (4)  $h(T, \beta) \leq h(T, \alpha) + H(\beta | \alpha)$ .
- (5)  $h(T, \alpha) = h(T, T^{-1}\alpha)$ .
- (6)  $h(T, \alpha) = h(T, \bigvee_{i=0}^{k-1} T^{-i}\alpha), \forall k \in \mathbb{Z}^+$ .
- (7) If  $T$  is invertible, then  $h(T, \alpha) = h(T, \bigvee_{i=-k}^k T^i\alpha), \forall k \in \mathbb{Z}^+$ .
- (8)  $h(T^k) = kh(T), \forall k \in \mathbb{Z}^+$ .
- (9) If  $T$  is invertible, then  $h(T^k) = |k|h(T), \forall k \in \mathbb{Z}$ .