Markov Chains and Dynamical Systems, Fall 2025

Sample problems for the First Quiz

1. Two cards are drawn (without replacement) from a usual deck of 52 cards. Let

$$A = \{\text{The first card drawn is a spade.}\} \qquad B = \{\text{The second card drawn is a spade.}\}$$

   Compute (i) $\mathbb{P}(A)$, (ii) $\mathbb{P}(B|A)$, (iii) $\mathbb{P}(B \cap A)$.

2. Two fair dice (a blue and a red die) are rolled. Consider

$$E_1 = \{\text{The red die turns up 5.}\}$$
$$E_2 = \{\text{The blue die turns up 2.}\}$$
$$E_3 = \{\text{The sum of the two values rolled is 7.}\}$$

   Are the events E_1, E_2 and E_3 pairwise independent? Are they independent as a collection?

3. We keep flipping a fair coin until it turns up Head for the first time. Let X denote the number of flips. What is the probability that X is even?

4. Let Y be a random variable that takes positive integer values and let

$$\mathbb{P}(Y \geq k) = \frac{1}{k!}$$

   Determine $\mathbb{E}Y$.