Markov Chains and Dynamical Systems, Fall 2025

Log: a brief summary of the classes

September 11

Introduction:

Dynamical systems: phase space M , map $T : M \to M$. Orbit, periodicity, asymptotic behavior. Simple examples on $M = \mathbb{R}$ or $M = [0, 1)$ with regular vs. chaotic patterns. Billiards.

Markov chains: state space, transition probabilities. A simple example: the weather chain. Stationary distribution and its relation to the following questions: on the average, what is the long term proportion of sunny days? If today it is sunny, what is the expected number of days that it is sunny again? Formula for the stationary distribution for chains with two states. *Random walk* on $\mathbb{Z}^2$.

Relation of the two topics: evolution at different time scales.

Gambler's ruin problem. Comparison of the gambler's ruin and the weather chain examples.

September 12

Probability recap: $(\Omega, \mathcal{F}, \mathbb{P})$ - sample spaces, σ-algebra of events, axioms of probability. Examples: roll two dice, infinite sequences of coin tosses. Inclusion-exclusion formula.

Conditional probability. Multiplication rule. Independence. Pairwise and complete independence of events $A_1, A_2, \ldots$

Discrete random variables: mass function, expected value $\mathbb{E}X$. The binomial and the geometric distributions. Two methods for computing $\mathbb{E}X$ – sums of indicators and $\sum_{k\geq 1} \mathbb{P}(X \geq k)$.

Independent random variables, independent sequences.

Definition of a (time homogeneous)*Markov chain.*