TRY-ON EXAM

Calculus 1.   December 18., 2023

Physicist - Engineers   Time for work: 90 minutes

Name:

Neptun code: ☐☐☐☐☐☐   **Practise Course Leader:**

1.	**2.**	**3.**	**4.**	**5*.**	Σ

Total score: 50 + 10

1. (**12**) Investigate and graph the following function

$$f(x) = \frac{x(x^2 + 1)}{x^2 - 1}.$$

2. (**8**) One side of an open field is bounded by a straight river. Determine how to put a fence around the other sides of a rectangular plot in order to enclose as great an area as possible with 2000 meters of fence.

3. (**4*5**) Determine the following integrals.

   (a)
   $$I_1 = \int \frac{x - 5}{(x + 1)(x - 2)^2} \mathrm{d}x$$

   (b)
   $$I_2 = \int \frac{\mathrm{d}x}{x(1 - \sqrt[3]{x})}$$

   (*Hint:* use the substitution $t = \sqrt[3]{x}$.)

   (c)
   $$I_3 = \int_2^3 \ln^2 x \, \mathrm{d}x$$

   (d)
   $$I_4 = \int_0^\infty \frac{2}{x^2 + 8x + 12} \mathrm{d}x$$

4. (**10**) Consider the region bounded by $x^2 = 4y$ and $y = \frac{x}{2}$. Find the volume generated by revolving the given region

   (a) about x-axis,

   (b) about y-axis.

5. (**More challenging task for extra score 10**) Find the surface area generated when the arc of $y = \ln x$, $1 \le x \le 7$ is revolved about the y-axis.