Functional Analysis,
Minimum requirements

Knowledge of the following definitions and theorems is a prerequisite for the exam.

1. **Definitions.**
   Norm, normed spaces, eqivalent norms. Banach space. Hamel basis, Schauder basis. Separability. Algebraic and topological dual spaces. Norm of functionals. Compact, sequentially compact, precompact subsets. Locally compact space. Equicontinous subset of continous functions. Subsets of first and second category (meager and nonmeager). Open map, graph of an operator. Closed and closable operator. Operator norm. Sublinear functional. Bidual. reflexive space. Weak and weak * convergence. Inner product. Hilbert space. Orthogonal complement. Orthonormal system. Compact operator. Adjoint of a bounded linear operator. Normal, unitary, selfadjoint operator. Partial isometry. Projections. Compact operator. Finite rank operator. Resolvent set and resolvent operator. Spectrum and its subdivision (point, continous, residual).

2. **Theorems.**
   Riesz's lemma. Arzela-Ascoli Theorem. Neumann series. Hahn-Banach theorems. Hölder inequality. The Riesz representation theorem. Baire Category Theorem. Uniform boundedness (Banach-Steinhaus) theorem. The open mapping theorem. Inverse mapping theorem. The closed graph theorem. Cauchy-Schwarz inequality. Parallelogram rule. Projection Theorem. Riesz's representation (Riesz-Fréchet) theorem. Sesquilinear forms. Gram matrices. Orthonormal systems. Gram-Schmidt process. Bessel's equality and inequality. Parseval's equality. Hellinger-Toeplitz theorem. Riesz-Schauder theorem.