

Functional Analysis, Course Material

I can ask you to prove the items **highlighted in bold** in the exam.

1. *Normed spaces.*

The existence of Hamel basis. Equivalent norms. Banach spaces with examples. Separability. Schauder basis. **Riesz's lemma**. Compactness, locally compact normed spaces. Arzela-Ascoli Theorem. Quotient spaces.

2. *Bounded linear operators on Banach spaces.*

Operator norm. **The equivalence of boundedness and continuity**. Integral operators. Invertible operators. **Neumann series**.

3. *Dual spaces*

Algebraic and topological dual spaces. Norm of functionals. Hahn-Banach theorems. Separation theorems. Minkowski functional. **Hölder inequality**. ℓ_p spaces. The dual space of continuous functions. The Riesz representation theorem. Positive linear functionals.

4. *Baire Category Theorem and its consequences.*

Subsets of first and second category. **Baire's theorem**. Uniform boundedness (Banach-Steinhaus) theorem. The open mapping theorem. Inverse mapping theorem. The closed graph theorem.

5. *Reflexive spaces*

The bidual of normed spaces, the canonical embedding into the bidual, reflexivity. Weak convergence and its properties. Mazur's theorem. Weak and weak* topology. Banach-Alaoglu theorem.

6. *Hilbert spaces.*

Cauchy-Schwarz inequality. Parallelogram rule, polarization formula. **Projection Theorem**. Best approximation. **Riesz's representation (Riesz-Fréchet) theorem**. Sesquilinear forms. Gram matrices. Orthonormal systems. Gram-Schmidt process. Bessel's equality and inequality. Parseval's equality. Separable Hilbert spaces.

7. *Bounded linear operators on Hilbert spaces.*

Operator norms. Adjoint of operators. The properties of the adjoint. Normal, unitary and selfadjoint operators. **Hellinger-Toeplitz theorem.** Positive operators. Projections. Compact operators and their properties. Finite rank operators.

8. *The spectrum of operators.*

The resolvent set and the resolvent operators. **Resolvent identities.** The spectrum and its subdivision (point, continuous, residual). The spectrum of unitary and selfadjoint operators. The spectrum of compact operators. Riesz-Schauder theorem.