

Functional Analysis 1
Sample Exam - Part II.
2024. május 29.

Name:

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Informations:

1. Working time: 60 minutes.
2. No aids of any kind may be used.

1. **(20 points)**

State and prove the Baire's Theorem.

2. **(15 points)**

Consider the right shift operator

$$S_R : (x_1, x_2, x_3, \dots) \mapsto (0, x_1, x_2, x_3, \dots)$$

as $\ell_2 \rightarrow \ell_2$ operators. Determine its adjoint and find its point-, continuous- and residual spectrum.

3. **(15 points)**

Let X be a Banach space, $T \in \mathcal{B}(X)$. Show that $\text{Ran } T$ is not dense iff there exists a nonzero $\varphi \in X^*$ s.t. $\varphi(T) = 0$.