Functional Analysis 1

Sample exam - Part I.

20 december 2024

Name:

Neptun code:

I.	II.	$\sum$:

Informations:

1. Working time: 60 minutes.
2. No aids of any kind may be used.
3. In the Minimum Requirements please state definitions and theorems as given in the lecture. **To pass the exam, a minimum of 24 points must be obtained in the Minimum Requirements.**
4. No justification is required in the Test.

(12×3 p.) **I. Minimum Requirements**

1. When do we say that two norms are equivalent?

2. What does the Arzela-Ascoli theorem state?

3. Define the operator norm.

4. What does it mean that a set is of first category (or meager)?

5. State the Baire's Theorem.

6. What does the Closed Graph Theorem assert?

7. State the Hahn-Banach Theorem.

8. What does the Hölder inequality assert?

9. What does it mean that a sequence is weakly convergent in a Banach space?

10. Give the definition of the spectrum of a bounded linear operator and define its parts.

11. When do we say that a linear operator is compact?

12. State the Riesz-Schauder Theorem.

(14×1 p.) **II. True or False Test.** Circle the letter T or F in front of the statements according to whether the statement is true or false.

1. T F All norms on $\mathbb{C}^n$ are equivalent.
2. T F ℓ_p is separable for all $1 \le p \le \infty$.
3. T F For an infinite series in a Banach space absolute convergence implies convergence.
4. T F In an infinite dimensional normed space the closed unit ball can be compact.
5. T F If a linear operator between normed spaces is bounded, then it is continous in 0.
6. T F The dual of ℓ_1 is ℓ_∞, but the dual of ℓ_∞ is not ℓ_1.
7. T F A complete metric space is always a countable union of nowhere dense closed subsets.
8. T F If a family of bounded linear operators between normed spaces is pointwise bounded, then it is uniformly bounded.
9. T F An open map maps closed sets to closed sets.
10. T F Every bounded operator between Banach spaces is closed.
11. T F Let X be an infinite dimensional normed space. If U is a weak open set in X, then it is bounded.
12. T F Every Banach space is a Hilbert space by the Polarization formula.
13. T F If A is a bounded and B is a compact operator, then AB is compact.
14. T F The resolvent set of a linear bounded operator is always a nonempty closed set in $\mathbb{C}$.