1. We roll two fair dice. Let $A$ denote the event that the sum of the two rolls are 6. Let $B$ denote the event that the first roll is 4. Show that $A$ and $B$ are not independent. Let $C$ denote the event that the sum of the two rolls is 7. Is $A$ independent from $C$?

2. We flip a fair coin three times. Let $A$ denote the event that the first flip is heads. Let $B$ denote the event that there are more heads than tails from the three flips. Calculate $P(B)$ and $P(B|A)$.

3. Dennis has 2 identically-looking dice, one of which is fair (it gives the numbers 1, 2, 3, 4, 5 and 6 with probability $\frac{1}{6} - \frac{1}{6}$ each), but the other one is loaded: 6 has a probability of $\frac{1}{2}$. Dennis picks one of them at random and rolls it twice. What is the probability that he rolls two sixes? What is the conditional probability of the event that he picked the loaded die, assuming he rolls two sixes?

4. A test for a certain disease works the following way: if the subject has the disease, it will be positive all the time; however, if the subject does not have the disease, the test will still be positive with probability 1%. In the entire population, 1 in 10000 people have this disease. What is the conditional probability of somebody actually having the disease assuming that his test was positive?

5. (a) We know the Smith family has two children, but we do not know how many of them are boys or girls. Assuming that at least one of their children is a girl, what is the probability that both are girls?
(b) We know the Smith family has two children, but we do not know how many of them are boys or girls. After knocking on their door, a girl opens the door. What is the probability that the other child is a girl as well?

6. A miner gets lost in the mine. He is in a chamber with 5 doors. Door 1 leads to a tunnel to the exit after 2 hours of walking. Door 2 leads to a tunnel to Door 3 after 1 hour of walking. Door 4 leads to a tunnel to Door 5 after 3 hours of walking. The miner picks a door at random, goes through the tunnel, but whenever he gets back to the chamber, he forgets his previous choices, and picks one of the five doors at random again.

$X$ denotes the total time it takes him to get to the exit. Calculate $E(X)$. (Hint: let $B_2$ denote the event that he picks Door 2 first. Argue that $E(X|B_2) = E(X) + 1$. Use total probability.)

7. We throw a fair coin 5 times. What is the probability of getting two heads?

8. We start rolling a regular 6-sided die. Let $X$ denote the total number of rolls until we get a 6, including the 6. Calculate the distribution of $X$. Let $Y$ denote the total number of rolls until we get a 6, not including the 6. Calculate the distribution of $Y$.

9. Let $X$ denote the total number of rolls needed to get a 6 with a regular 6-sided die. What is the distribution of $X$? Assuming the first roll is not a 6, what is the conditional distribution of the additional number of rolls needed to get a 6? (This is called the memoryless property of the geometric distribution.)

10. A test has 20 yes or no questions. For each question, we know the correct answer with probability $\frac{5}{7}$, we are convinced of the wrong answer with probability $\frac{1}{7}$. If we don’t know the answer, we guess yes or no with probability $\frac{1}{2} - \frac{1}{2}$. What is the probability of giving a correct answer for the first question? What is the distribution of the number of correct answers? What is the probability of giving at least 18 correct answers?

11. There is an average of 2.3 shark attacks registered at the beaches of Florida each year. What is the probability that in a given year, at most 1 shark attack occurs?

12. A book with 500 pages contains 500 typos. What is the probability that on a random page there are at least 2 typos? (We assume that each typo appears on every page with the same probability, and independently from other typos.)

13. Assume that a web server has on average 5 arrivals per minute. What is the probability that during a 30 second interval, there are at least 3 arrivals?
14. Let $X$ denote the value of a roll with a fair six-sided die. Calculate the mean and variance of $X$.

15. At a sports competition, participants have to throw a ball as far as possible. Let $X$ denote the result of Jane (in meters). $X$ has the following probability density function:

\[
f(x) = \begin{cases} \frac{75}{30} & 30 \leq x \leq 50 \\ 0 & \text{otherwise} \end{cases}
\]

(a) Calculate the probability that Jane throws further away than 45 meters.
(b) Calculate the cumulative distribution function of $X$.
(c) Calculate $E(X)$.
(d) Each participant can throw the ball 3 times, and their score is the maximum of the 3 throws. Calculate the distribution of Jane’s score (we assume that different throws are independent).

16. A thermometer works the following way: if the real temperature is $x$ degrees, then the thermometer will display a uniform random value between $x - 1$ and $x$. To counteract this, the temperature is measured 5 times, then the largest value is used. What is the probability that the obtained measurement differs from the real temperature by more than 0.2 degrees?

17. Assume that the age of a light bulb $X$ (measured in 100 hours) has an exponential distribution such that $P(X > 10) = 0.8$. Calculate the parameter of the exponential distribution and the mean of $X$.

18. In a given population, the height of the members has average 177 cm and deviation 6 cm. What is the probability that a member picked at random has height over 190 cm?

19. In a class of 120 students, Stochastics and Calculus marks are as follows:

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We pick a student at random; let $X$ denote his Stochastics mark and $Y$ his Calculus mark.

(a) $P$(the student failed from at least one of the courses) =?
(b) $E(X)$ =?
(c) $E(X|Y \geq 4)$ =?
(d) Are $X$ and $Y$ independent?
(e) $\text{cov}(X, Y)$ =? (Bonus question: how were the numbers in the table designed?)