Stochastics
Problem sheet 1 - Basic probability 1
Fall 2021

1. We roll two fair dice. Let A denote the event that the sum of the two rolls are 6. Let B denote the event that the first roll is 4. Show that A and B are not independent. Let C denote the event that the sum of the two rolls is 7. Is B independent from C?

2. We flip a fair coin three times. Let A denote the event that the first flip is heads. Let B denote the event that there are more heads than tails from the three flips. Calculate $\mathbf{P}(B)$ and $\mathbf{P}(B|A)$.

3. Dennis has 2 identically-looking dice, one of which is fair (it gives the numbers 1, 2, 3, 4, 5 and 6 with probability $\frac{1}{6} - \frac{1}{6}$ each), but the other one is loaded: 6 has a probability of $\frac{1}{2}$. Dennis picks one of them at random and rolls it twice. What is the probability that he rolls two sixes? What is the conditional probability of the event that he picked the loaded die, assuming he rolls two sixes?

4. A test for a certain disease works the following way: if the subject has the disease, it will be positive all the time; however, if the subject does not have the disease, the test will still be positive with probability 1%. In the entire population, 1 in 10000 people have this disease. What is the conditional probability of somebody actually having the disease assuming that his test was positive?

5. (a) We know the Smith family has two children, but we do not know how many of them are boys or girls. Assuming that at least one of their children is a girl, what is the probability that both are girls?
   (b) We know the Smith family has two children, but we do not know how many of them are boys or girls. After knocking on their door, a girl opens the door. What is the probability that the other child is a girl as well?

6. A miner gets lost in the mine. He is in a chamber with 5 doors. Door 1 leads to a tunnel to the exit after 2 hours of walking. Door 2 leads to a tunnel to Door 3 after 1 hours of walking. Door 4 leads to a tunnel to Door 5 after 3 hours of walking. The miner picks a door at random, goes through the tunnel, but whenever he gets back to the chamber, he forgets his previous choices, and picks one of the five doors at random again.

   X denotes the total time it takes him to get to the exit. Calculate $\mathbf{E}X$. (Hint: let B_2 denote the event that he picks Door 2 first. Argue that $\mathbf{E}(X|B_2) = \mathbf{E}(X) + 1$. Use total probability.)

7. At a sports competition, participants have to throw a ball as far as possible. Let X denote the result of a single throw of Jane (in meters). X has the following probability density function:

$$f(x) = \begin{cases} \frac{75}{x^2} & 30 \le x \le 50 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Calculate the probability that Jane throws further away than 45 meters.
   (b) Calculate the cumulative distribution function of X.
   (c) Calculate $\mathbf{E}X$.
   (d) Each participant can throw the ball 3 times, and their score is the maximum of the 3 throws. Calculate the distribution of Jane's score (we assume that different throws are independent).

8. A thermometer works the following way: if the real temperature is x degrees, then the thermometer will display a uniform random value between $x - 1$ and x. To counteract this, the temperature is measured 5 times, then the largest value is used. What is the probability that the obtained measurement differs from the real temperature by more than 0.2 degrees?

HW1. (Deadline: 21 Sept.) A shooting gallery has 6 guns. Three of them are such that we hit the target with probability 0.5, with one, the probability of hitting the target is 0.7 and with two, the probability of hitting the target is 0.8. We pick a gun at random, then shoot. What is the probability of hitting the target? What is the conditional probability of choosing a 0.8 gun assuming that we hit the target?