Stochastics
Problem sheet 10 - Statistics II: tests for the mean
Fall 2021

1. We can only measure the inner diameter of a tube with error. If the tube has inner diameter $d$, then each measurement will have mean $d$ and deviation $\sigma = 0.2$ millimeter. We measure the diameter 5 times to obtain the sample $10.1, 10.2, 10.0, 10.1, 9.9$ (millimeter). Test the hypothesis that the inner diameter is $d = 10$ (millimeter) against the hypothesis that $d \neq 10$ on a 95% confidence level.

2. A company produces cement in packs with 25kg nominal weight. Due to the packaging process, the amount of cement in a single pack has deviation 0.5kg, but the expectation $\mu$ is unknown. We examine 25 packs, and the mean of the cement inside turns out to be 24.84kg.

   (a) Do we accept the hypothesis $H_0$ that $\mu = 25$ against the hypothesis $H_1$ that $\mu \neq 25$ with a confidence level 95%?

   (b) Do we accept the hypothesis $H_0$ that $\mu = 25$ against the hypothesis $H_1$ that $\mu \neq 25$ with a confidence level 90%?

   (c) Assume the deviation for each pack is only 0.3kg. Do we accept the hypothesis $H_0$ that $\mu = 25$ against the hypothesis $H_1$ that $\mu \neq 25$ with a confidence level 95%?

3. We can only measure the height of a mountain with error. If the mountain has height $h$, then each measurement will have mean $h$ and deviation $\sigma = 3$ meters. We measure the height 6 times to obtain the sample $3845, 3853, 3848, 3845, 3846$ (meters). Test the hypothesis that the height is $h = 3851$ (meters) against the hypothesis that $h \neq 3851$ on a 95% confidence level.

4. We measure the concentration of salt in a dilution. We obtain the following sample after 5 measurements: (g/l): 7.7, 8.1, 7.7, 7.5, 7.0. Previously, someone stated that the concentration is 7 g/l. Do we accept this on a 95% confidence level against the hypothesis that the concentration is not equal to 7 g/l? And what about the following sample: 7.5, 7.4, 7.3, 7.4, 7.5?

5. A company wants to motivate its employees to increase productivity. The company tests two different methods: method A is to increase the salary of people, and method B is to improve the work environment. The change in productivity was measured for all 6 employees with both methods:

   | employee | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|
   | work env. impr. | 1.2 | 1.0 | 0.8 | 0.6 | 0.9 | 0.9 |
   | salary incr. | -0.2 | 0.3 | 3.6 | 1.4 | -0.1 | 1.6 |

   (a) Test on a 95% confidence level whether improving the work environment increases productivity or not. (What is the null hypothesis?)

   (b) Test on a 95% confidence level whether increasing the salary increases productivity or not.

   (c) Test on a 95% confidence level whether increasing the salary increases productivity more than improving the work environment.

6. We test a blood pressure drug. We measure the decrease of blood pressure in 5 patients and obtain the sample $4.0, 8.0, 5.0, 7.0, 1.0$. Test the hypothesis that the drug is ineffective against the hypothesis that it has a positive effect on a 99% confidence level.

7. Two different types of chicken feed are tested. Feed A was tested on a group of 5 chicken and feed B was tested on a different group of 6 chicken, with the weight increase of each chicken measured:

   | A | 1 | 2 | 3 | 4 | 5 |
   |---|---|---|---|---|---|
   | | 1.4 | 1.3 | 1.6 | 2.1 | 1.6 |

   | B | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|
   | | 4.1 | 3.3 | -1.1 | 0.1 | 4.3 | 0.7 |

   The producer of feed B claims that feed B increases the weight of chicken more than feed A. Test the hypothesis that the two feeds provide the same increase to the weight of chicken against the hypothesis that feed B increases the weight more on a 95% confidence level.

HW7. (Deadline: 7 Dec.) We measure the unknown alcohol content of a wine. Our measuring method has an error with 0 mean and 0.5% deviation. We make 5 measurements with the results

$$12.6\%, 12.8\%, 12.6\%, 12.9\%, 12.4\%.$$

Do we accept the hypothesis that the alcohol content is 12.5% against the hypothesis that the alcohol content is higher than 12.5% with a confidence level of 95%?