Stochastics
Problem sheet 1 - Basic probability 1, solutions
Fall 2021

2. We flip a fair coin three times. Let A denote the event that the first flip is heads. Let B denote the event that there are more heads than tails from the three flips. Calculate $\mathbf{P}(B)$ and $\mathbf{P}(B|A)$.

Result. $\mathbf{P}(B) = \frac{1}{2}$ and $\mathbf{P}(B|A) = \frac{3}{4}$.

4. A test for a certain disease works the following way: if the subject has the disease, it will be positive all the time; however, if the subject does not have the disease, the test will still be positive with probability 1%. In the entire population, 1 in 10000 people have this disease. What is the conditional probability of somebody actually having the disease assuming that his test was positive?

Result. $100/10099 \approx 0.0099$ (so just under 1%).

5. (a) We know the Smith family has two children, but we do not know how many of them are boys or girls. Assuming that at least one of their children is a girl, what is the probability that both are girls?

   (b) We know the Smith family has two children, but we do not know how many of them are boys or girls. After knocking on their door, a girl opens the door. What is the probability that the other child is a girl as well?

Results.

(a) There are 4 possibilities:

younger	older	prob.
boy	boy	1/4
boy	girl	1/4
girl	boy	1/4
girl	girl	1/4

Then
$$\mathbf{P}(\text{two girls} \mid \text{at least one girl}) = \frac{\mathbf{P}(\text{two girls})}{\mathbf{P}(\text{at least one girl})} = \frac{1/4}{3/4} = 1/3.$$

(b) There are two sources of randomness: whether the children are boys or girls, and who opens the door. So there are 8 possible outcomes:

younger	older	who opens	prob.
boy	boy	younger	1/8
boy	boy	older	1/8
boy	girl	younger	1/8
boy	girl	older	1/8
girl	boy	younger	1/8
girl	boy	older	1/8
girl	girl	younger	1/8
girl	girl	older	1/8

The following outcomes are included in the event that a girl opens the door:

younger	older	who opens	prob.
boy	girl	older	1/8
girl	boy	younger	1/8
girl	girl	younger	1/8
girl	girl	older	1/8

Out of these outcomes, 2 are such that the other child is also a girl, so the conditional probability that the other child is a girl as well assuming a girl opens the door is $\frac{2/8}{4/8} = 1/2$.

8. A thermometer works the following way: if the real temperature is x degrees, then the thermometer will display a uniform random value between $x - 1$ and x. To counteract this, the temperature is measured 5 times, then the largest value is used. What is the probability that the obtained measurement differs from the real temperature by more than 0.2 degrees?

Result. $0.8^5 \approx 0.33$. Note that the value of x is not relevant to the problem.