1. We roll two fair dice. Let $E_1$ denote the event that the sum of the two rolls are 6. Let $F$ denote the event that the first roll is 4. Show that $E_1$ and $F$ are not independent. Let $E_2$ denote the event that the sum of the two rolls is 7. Is $E_2$ independent from $F$?

2. A shooting gallery has 6 guns. Three of them are such that we hit the target with probability $0.5$, with one, the probability of hitting the target is $0.7$ and with one, the probability of hitting the target is $0.8$. We pick a gun at random, then shoot. What is the probability of hitting the target? What is the conditional probability of choosing a 0.8 gun assuming that we hit the target?

3. Quality control at the clamp factory. Three large crates of clamps come out of the factory

4. A test for a certain disease works the following way: if the subject has the disease, it will be positive all the time; however, if the subject does not have the disease, the test will still be positive with probability 1%. In the entire population, 1 in 10000 people have this disease. What is the conditional probability of somebody actually having the disease assuming that his test was positive?

Solution. We test someone. $A$ is the event that the test is positive, $B_1$ is the event that he has the disease and $B_2$ is the event that he does not have the disease. Then, using total probability

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) = 1 \cdot \frac{1}{10000} + \frac{1}{100} \cdot \frac{9999}{10000} = 0.010099.$$ 

Using Bayes theorem, we get

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A)} = \frac{1 \cdot 1/10000}{0.010099} = 0.00990,$$

so even if the test is positive, there is only approximately 1% chance that the person has the disease.

5. We throw a fair coin 5 times. What is the probability of getting two heads?

6. A test has 20 yes or no questions. For each question, we know the correct answer with probability $\frac{5}{7}$, we are convinced of the wrong answer with probability $\frac{1}{7}$. If we don’t know the answer, we guess yes or no with probability $\frac{1}{2} - \frac{1}{2}$. What is the probability of giving a correct answer for the first question? What is the distribution of the number of correct answers? What is the probability of giving at least 18 correct answers?

7. A thermometer works the following way: if the real temperature is $x$ degrees, then the thermometer will display a uniform random value between $x - 1$ and $x$. To counteract this, the temperature is measured 5 times, then the largest value is used. What is the probability that the obtained measurement differs from the real temperature by more than 0.2 degrees?

8. Assume that the age of a light bulb $X$ (measured in 100 hours) has an exponential distribution such that $P(X > 10) = 0.8$. Calculate the parameter of the exponential distribution and the mean of $X$.

9. In a given population, the height of the members has average 177 cm and deviation 6 cm. What is the probability that a member picked at random has height over 190 cm?

10. A book with 500 pages contains 500 typos. What is the probability that on a random page there are at least 2 typos? (We assume that each typo appears on every page with the same probability, and independently from other typos.)

11. There is an average of 2.3 shark attacks registered at the beaches of Florida each year. What is the probability that in a given year, at most 1 attack occurs?

12. Let $X$ denote the value of a roll with a fair six-sided die. Calculate the mean and deviation of $X$.

13. Assume that a web server has on average 5 arrivals per minute. What is the probability that during a 30 second interval, there are at least 3 arrivals?

HW1. (Deadline: 19 Sept.) Dennis has 2 identically-looking dice, one of which is fair (it gives the numbers 1, 2, 3, 4, 5 and 6 with probability $\frac{1}{6} - \frac{1}{6}$ each), but the other one is loaded: 6 has a probability of $\frac{1}{2}$, the other numbers have probability $\frac{1}{10} - \frac{1}{10}$ each. Dennis picks one of them at random and rolls it twice. What is the probability that he rolls two sixes? What is the conditional probability of the event that he picked the loaded die, assuming he rolls two sixes?