Stochastics
Problem sheet 2 - Basic probability 2
Fall 2021

1. We throw a fair coin 5 times. What is the probability of getting two heads?

2. We start rolling a regular 6-sided die. Let X denote the total number of rolls until we get a 6, including the 6. Calculate the distribution of X. Let Y denote the total number of rolls until we get a 6, not including the 6. Calculate the distribution of Y.

3. Let X denote the total number of rolls needed to get a 6 with a regular 6-sided die. What is the distribution of X? Assuming the first roll is not a 6, what is the conditional distribution of the additional number of rolls needed to get a 6? (This is called the memoryless property of the geometric distribution.)

4. A test has 20 yes or no questions. For each question, we know the correct answer with probability $\frac{5}{7}$, we are convinced of the wrong answer with probability $\frac{1}{7}$. If we don't know the answer, we guess yes or no with probability $\frac{1}{2}-\frac{1}{2}$. What is the probability of giving a correct answer for the first question? What is the distribution of the number of correct answers? What is the probability of giving at least 18 correct answers?

5. There is an average of 2.3 shark attacks registered at the beaches of Florida each year. What is the probability that in a given year, at most 1 attack occurs?

6. A book with 500 pages contains 1000 typos. What is the probability that on a random page there are at least 2 typos? (We assume that each typo appears on every page with the same probability, and independently from other typos.)

7. Assume that a web server has on average 5 arrivals per minute. What is the probability that during a 30 second interval, there are at least 3 arrivals?

8. Assume that the age of a light bulb X (measured in 100 hours) has an exponential distribution such that $\mathbf{P}(X > 10) = 0.8$. Calculate the parameter of the exponential distribution and the mean of X.

9. In a given population, the height of the members has average 177 cm and deviation 6 cm. What is the probability that a member picked at random has height over 190 cm?

10. In a class of 120 students, Stochastics and Calculus marks are as follows:

$C\backslash S$	1	2	3	4	5
1	1	2	2	1	4
2	2	4	4	8	2
3	4	8	8	12	8
4	5	4	6	9	6
5	0	6	4	6	4

We pick a student at random; let X denote his Stochastics mark and Y his Calculus mark.

(a) $\mathbf{P}$(the student failed from at least one of the courses) =?

(b) $\mathbf{E}(X)$ =?

(c) $\mathbf{E}(X|Y \geq 4)$ =?

(d) Are X and Y independent?

(e) $\mathrm{cov}(X, Y)$ =? (Bonus question: how were the numbers in the table designed?)