Stochastics
Problem sheet 2 - Basic probability 2, solutions
Fall 2021

2. We start rolling a regular 6-sided die. Let X denote the total number of rolls until we get a 6, including the 6. Calculate the distribution of X. Let Y denote the total number of rolls until we get a 6, not including the 6. Calculate the distribution of Y.

   Result. The distribution of the number of rolls is GEO(1/6). The distribution of the number of rolls not including the six is PGEO(1/6).

3. Let X denote the total number of rolls needed to get a 6 with a regular 6-sided die. What is the distribution of X? Assuming the first roll is not a 6, what is the conditional distribution of the additional number of rolls needed to get a 6? (This is called the memoryless property of the geometric distribution.)

   Result. The distribution of the number of rolls is GEO(1/6). The conditional distribution of the number of additional rolls assuming the first roll is not a 6 is also GEO(1/6).

4. A test has 20 yes or no questions. For each question, we know the correct answer with probability $\frac{5}{7}$, we are convinced of the wrong answer with probability $\frac{1}{7}$. If we don't know the answer, we guess yes or no with probability $\frac{1}{2}$–$\frac{1}{2}$. What is the probability of giving a correct answer for the first question? What is the distribution of the number of correct answers? What is the probability of giving at least 18 correct answers?

   Result. The probability of a correct answer is $5/7 \cdot 1 + 1/7 \cdot 1/2 + 1/7 \cdot 0 = 11/14$ by total probability. The distribution of the number of correct answers is $\mathrm{BIN}(20, 11/14)$. The probability of at least 18 answers is $\binom{20}{18}\left(\frac{11}{14}\right)^{18}\left(\frac{3}{14}\right)^{2} + \binom{20}{19}\left(\frac{11}{14}\right)^{19}\left(\frac{3}{14}\right)^{1} + \binom{20}{20}\left(\frac{11}{14}\right)^{20}\left(\frac{3}{14}\right)^{0}$.

7. Assume that a web server has on average 5 arrivals per minute. What is the probability that during a 30 second interval, there are at least 3 arrivals?

   Result. Let X denote the number of arrivals during a 30 second interval. Then $X \sim \mathrm{POI}(2.5)$, and

$$\mathbf{P}(X \geq 3) = 1 - \mathbf{P}(X = 0) - \mathbf{P}(X = 1) - \mathbf{P}(X = 2) = 1 - \frac{2.5^0}{0!}e^{-2.5} - \frac{2.5^1}{1!}e^{-2.5} - \frac{2.5^2}{2!}e^{-2.5} \approx 0.456.$$

9. In a given population, the height of the members has average 177 cm and deviation 6 cm. What is the probability that a member picked at random has height over 190 cm?

   Result. $\int_{x=190}^{\infty} \frac{1}{\sqrt{2\pi}\cdot 6} e^{-\frac{(x-177)^2}{2\cdot 6^2}} \mathrm{d}x \approx 0.015$.