Problem sheet 3  
Poisson point process  
Fall 2017

1. A call center receives an average of 8 local and 2 long-distance calls during 5 minutes.
   (a) What is the probability that during 2 minutes, they receive exactly 1 long-distance call?
   (b) What is the probability that during 2 minutes, they receive at most 3 calls in total?
   (c) What is the conditional probability that during 2 minutes, they receive exactly 1 long-distance call, assuming that during the same period of time, they receive at most 3 calls in total?
   (d) We start logging calls at \( t = 0 \). What is the distribution and the mean of the time of the first local call?
   (e) What is the distribution and the mean of the time of the first call (of any type)?

2. On a one-way road, an average of 2 cars per minute pass. Dennis stands next to the road and throws rocks at the passing cars. For each passing car, he breaks the window with probability \( \frac{1}{4} \). Curly, who is standing further down the road, is counting cars with intact and broken glass. What is the probability that during a 1 minute interval, Curly counts at most 2 cars with a broken window and exactly 1 car with an intact window?

3. A certain type of cookies contain on average 3 chocolate chips per cookie and 2 raisins per cookie.
   (a) What is the probability that a random cookie will contain exactly 2 chocolate chips?
   (b) What is the probability that a random cookie will contain no raisins?
   (c) Assuming that a cookie contains a total of 2 pieces (of either chocolate chips or raisins), what is the conditional probability that both of them are chocolate chips?
   (d) Joe eats half of a cookie. What is the probability that it contains at least 1 raisin?
   (e) Joe eats the second half of the cookie too. What is the conditional probability that the entire cookie contains at least 2 raisins, assuming that the first half contained at least 1 raisin?

4. On a road, an average of 2 cars per minute pass by. Jack stands next to the road and starts counting cars.
   (a) What is the probability that during a 5 minute interval, no cars pass Jack?
   (b) What is the probability that during a 4 minute interval, at most 3 cars pass him by?
   (c) What is the probability that during a 2 minute interval, 2 cars pass him by, then during the next 2 minutes, no cars pass him by?
   (d) On average, 10% of the cars are red. What is the probability that during a 5 minute interval, no red car passes by?
   (e) What is the probability that during a 3 minute interval, exactly 1 red car and exactly 2 non-red cars pass by?

5. Two types of jobs arrive at a server: type A and type B. On average, the arrival rates are 1 job/second for type A and 2 jobs/second for type B. What is the probability that the first job arriving is of type A? What is the distribution of the waiting time before the first arrival?

6. Let \( T_0 \) denote the length of the interval containing 0 in a Poisson point process on the real line. Calculate \( E(T_0) \). Determine the distribution of \( T_0 \).

7. * Let \( X_n \) denote the \( n \)-th arrival after time 0 in a Poisson process with rate \( \lambda \). Calculate the distribution of \( X_n \) (that is, calculate the probability density function of \( X_n \)).

8. On average, an office receives a phone call every 5 minutes. For each call, the two operators, Alice and Bob flip a fair coin to decide who takes the call.
   (a) Calculate the probability that exactly two calls arrive between 10:00 and 10:20.
   (b) Calculate the probability that between 10:00 and 10:20, all calls are taken by Alice.
   (c) Assuming that between 10:00 and 10:20, all calls were taken by Alice, what is the conditional probability that she has taken exactly two calls?
9. Type A and type B packets arrive at a server. On average, 1 type A packet arrives per second, and 1.5 type B packets arrive per second.

(a) Calculate the probability that exactly 2 packets arrive over a 2 second interval.
(b) What is the conditional probability that during a 2 second interval, exactly 1 type A packet arrives, assuming 2 packets arrive in total?

10. In a forest, there are on average 10 trees per 100m². Let us assume that each tree has diameter 20 cm on the ground level. (Ignore the possibility that they may overlap.)

(a) What is the probability that there are no trees on a given 10m² area?
(b) We fire a bullet in a random direction from the middle of the forest. What is the probability that the bullet will fly at least 50 meters before it hits a tree?

11. We count trucks on a road. Truck traffic is inhomogeneous, the density of trucks during the day has rate function (number of trucks per hour):

\[ \lambda(t) = 6 - 4 \cos \left( \frac{\pi}{6} t \right) \quad t \in [0, 24] \]

(a) Plot the rate function. At what time are the maximum points?
(b) What is the average number of trucks passing through the road during one day?
(c) What is the probability that during 12:00 and 13:00, exactly 3 trucks pass?

HW3 (Deadline: 17 Oct.) Jon is receiving an average of 0.6 work-related e-mails per hour, an average of 0.4 personal e-mails per hour and an average of 1 spam e-mail per hour.

(a) Calculate the probability that between 10:00 and 18:00, he receives exactly 3 work-related e-mails.
(b) Calculate the probability that between 10:00 and 12:00, he receives exactly 4 e-mails in total.
(c) Calculate the probability that between 10:00 and 12:00, he receives exactly 4 e-mails in total, and none of them are spam.