

Stochastics  
Problem sheet 3 - Generating functions, some results  
Fall 2021

5. Let  $X_1, X_2, \dots$  be iid random variables and  $N$  a discrete random variable, independent from the  $X$ 's, and let  $Y = \max(X_1, \dots, X_N)$ . Express the cumulative distribution function of  $Y$  using the common cumulative distribution function of the  $X$ 's and the generating function of  $N$ .

Result. Compute  $\mathbb{P}(Y < x)$  with total probability according to the value of  $N$ ; if  $F(x)$  denotes the cdf of the  $X_i$ 's and  $G_N(z)$  denotes the probability generating function of  $N$ , then

$$\mathbb{P}(Y < x) = G_N(F(x)).$$