5. Let $X_1, X_2, \ldots$ be iid random variables and $N$ a discrete random variable, independent from the $X$’s, and let $Y = \max(X_1, \ldots, X_N)$. Express the cumulative distribution function of $Y$ using the common cumulative distribution function of the $X$’s and the generating function of $N$.

Result. Compute $\mathbb{P}(Y < x)$ with total probability according to the value of $N$; if $F(x)$ denotes the cdf of the $X_i$’s and $G_N(z)$ denotes the probability generating function of $N$, then

$$\mathbb{P}(Y < x) = G_N(F(x)).$$