1. A server has a buffer that contains a random number of jobs with distribution GEO(θ), but θ is unknown. We check the number of jobs in the buffer at 10 different times, and obtain the sample 1, 0, 2, 0, 0, 1, 0, 3, 0, 1.
   - Use the maximum likelihood method to give an estimate for θ.
   - Use the moment method to give an estimate for θ.

2. The income of people in a country is measured on a scale where $x = 1$ corresponds to minimum wage. We assume that the distribution of income can be described by the density function $f(x) = \frac{\theta}{x^2}$ ($x \geq 1$). (This is the so-called Pareto distribution.) Give a ML estimate on θ based on the following sample of 10 random people: 1.53, 2.76, 19.65, 4.16, 7.31, 1.21, 254.2, 5.45, 1.12, 1.63.

3. A sample of 5 values were taken from a uniform distribution on the interval $[0, \theta]$, where θ is unknown. The sample is 0.212, 0.255, 0.300, 0.165, 0.068.
   - Use the maximum likelihood method to give an estimate for θ.
   - Use the moment method to give an estimate for θ.

4. A sample of 5 values were taken from a uniform distribution on the interval $[0, \theta]$, where θ is unknown. The sample is 0.12, 0.08, 0.40, 0.05, 0.10. Use the moment method to give an estimate for θ. Explain the result.

5. * A discrete time Markov chain has two states: 1 and 2. The transition matrix is unknown, the initial vector is (1, 0). Based on the following sample from the Markov chain, give a ML estimate for the matrix $P$.

   $\begin{pmatrix}
   1, 2, 1, 2, 2, 1, 2, 2 \\
   \end{pmatrix}$

   Does the stationary distribution of ML estimate $\hat{P}$ coincide with the relative occurrence of each state in the sample? Explain the difference.

6. A company produces sugar in packs with 1000g nominal weight. Due to the packaging process, the amount of sugar in a single pack has deviation 50g, but the expectation $m$ is unknown. We examine 25 packs, and the mean of the sugar inside turns out to be 986 g. Do we accept the hypothesis $H_0$ that $m = 1000$ against the hypothesis $H_1$ that $m \neq 1000$ with a confidence level 95% (that is, $\epsilon = 0.05$)? What if the deviation was only 20g?

7. We measure the unknown alcohol content of a wine. Our measuring method has an error with 0 mean and 0.5% deviation. We make 5 measurements with the results 12.6%, 12.8%, 12.6%, 12.9%, 12.4%.
   (a) Give a maximum likelihood estimate for the alcohol content.
   (b) Do we accept the hypothesis that the alcohol content is 12.5% against the hypothesis that the alcohol content is higher than 12.5% with a confidence level of 95%?

8. We measure the concentration of salt in a dilution. We obtain the following sample after 5 measurements: (g/l): 7.7, 8.1, 7.7, 7.5, 7.0. Previously, someone stated that the concentration of the dilution is 7 g/l. Do we accept this on a 95% confidence level against the hypothesis that the concentration is not equal to 7 g/l?

9. A pharmaceutical company is doing test trials for its new blood pressure drug. They tested the drug on 6 people. Do we accept the hypothesis $H_0$ that the mean is $\mu = 0$ against the hypothesis $H_1$ that $\mu < 0$ with a confidence level of 95%, if the results for the 6 people are . . .
   (a) $-1.2, -3.1, -2.8, -0.2, +0.1, +1.2$?
   (b) $-0.6, -1.2, -0.8, -1.5, -1.4, -0.5$?

   (Note that $H_0$ corresponds to the drug being ineffective.) What if the confidence level is 99.9% instead?