7. We are measuring the height of a mountain, but we can only measure it with an additive error with \( \sigma = 10 \) in meters (so if the real height is \( h \), then the measurement has distribution \( N(h, 10^2) \)). Give a moment estimate for the height of the mountain based on the following 7 measurements: 3546, 3550, 3548, 3562, 3537, 3550, 3557.

Solution. \( X_i \sim N(h, 10^2) \), and the mean of \( X_i \) is \( g(h) = E_h(X_1) = h \), so \( g \) is the identity function, and the moment estimator for \( h \) is \( g^{-1}(\bar{x}) = \bar{x} = 3550 \).

10. We measure the concentration of salt in a dilution. We obtain the following sample after 5 measurements: (g/l): 7.7, 8.1, 7.7, 7.5, 7.0. Previously, someone stated that the concentration is 7 g/l. Do we accept this on a 95% confidence level against the hypothesis that the concentration is not equal to 7 g/l? And what about the following sample: 7.5, 7.4, 7.3, 7.4, 7.5?

Solution. This is a 1-sample, 2-sided t-test to decide between

- \( H_0: c = 7 \) against
- \( H_1: c \neq 7 \).

For the first sample, \( \bar{x} = 7.6, s_n^* = 0.4 \), and

\[
t = \frac{\bar{x} - c}{s_n^*} \sqrt{n} = \frac{7.6 - 7}{0.4} \sqrt{5} = 3.354,
\]

while \( t_{\epsilon/2} \) is the two-tail, 0.05 quantile of the t-distribution with degree of freedom \( n - 1 = 4 \), which is

\[ t_{\epsilon/2} = 2.776. \]

\(-t_{\epsilon/2} < t < t_{\epsilon/2}\) does not hold, so we accept \( H_1 \).

For the second sample, \( \bar{x} = 7.4, s_n^* = 0.0866 \), and

\[
t = \frac{\bar{x} - c}{s_n^*} \sqrt{n} = \frac{7.4 - 7}{0.0866} \sqrt{5} = 10.328.
\]

\(-t_{\epsilon/2} < t < t_{\epsilon/2}\) does not hold, so we accept \( H_1 \) in this case, too.