Stochastics
Problem sheet 9 - Statistics 1: parameter estimation
Fall 2021

1. We have a (possibly loaded) six-sided die where the probability p of rolling a 6 is unknown. We roll 10 times and get the numbers 3, 6, 5, 6, 1, 4, 2, 6, 6, 4. Give a maximum likelihood estimate for p.

2. We have a (possibly loaded) six-sided die where the probability p of rolling a 6 is unknown. Out of 10 rolls, we get 4 sixes. Give a maximum likelihood estimate for p. Give a moment estimate for p.

3. The income of people in a country is measured on a scale where $x = 1$ corresponds to minimum wage. We assume that the distribution of income can be described by the density function $f(x) = \frac{\theta}{x^{\theta+1}}$ $(x \geq 1)$. (This is the Pareto distribution.) Give a ML estimate on θ based on the following sample of 10 random people: $1.53, 2.76, 19.65, 4.16, 7.31, 1.21, 254.2, 5.45, 1.12, 1.63$.

4. * A lake contains N fish, where N is unknown. They catch and mark 50 fish which are then let back in the lake. Some time later, they catch 40 fish from the lake, and find that 4 of them are marked. Give a moment estimate for N. Give a maximum likelihood estimate for N.

5. A sample of 5 values were taken from a uniform distribution on the interval $[0, a]$, where a is unknown. The sample is $0.212, 0.255, 0.300, 0.165, 0.068$.

    (a) Give the moment estimate for a.

    (b) Give the ML estimate for a. (Take into account that the likelihood function is not continuous.)

6. A sample of 5 values were taken from a uniform distribution on the interval $[0, a]$, where a is unknown. The sample is $0.12, 0.08, 0.40, 0.05, 0.10$. Compute the moment estimator for a. Explain the result.

7. Historically, 60% of students pass the exam of a certain course. Last semester, 14 students passed the exam, but the N number of students who took the exam is unknown. Give a ML estimate for N. Can we give a moment estimate for N?

8. In an M/M/1 queue, the number of jobs in the buffer has distribution PGEO$(1 - \rho)$, where ρ is the load of the queue $(0 < \rho < 1)$. We check the number of jobs in the queue at 5 different points in time, and obtain the sample 2, 0, 4, 1, 1. Give a maximum likelihood estimate for the load of the queue.

9. Cars are passing by on a road with independent exponentially distributed interarrival times. The parameter λ of the exponential distribution is unknown (λ is measured in 1/sec). We register the following interarrival times (in seconds): 8, 15, 4, 13, 35. Calculate the moment estimate for λ based on the sample.

10. * A discrete time Markov chain has two states: 1 and 2. The transition matrix is unknown, the initial vector is (1 0). Based on the following sample from the Markov chain, give a ML estimate for the matrix P.
$$1, 2, 1, 1, 2, 2, 2, 1, 2, 2$$

Does the stationary distribution of the ML estimator $\hat{P}$ coincide with the relative occurrence of each state in the sample? Explain the difference.