1. We have 2 large boxes of screws. Each screw can be proper (good quality) or scrap (bad quality). In one of the boxes, 98% of the screws are proper, while in the other box, only 96% of the screws are proper, but we don’t know which box is which. We take out 10 screws from one of the boxes randomly, and check them.

(a) What is the probability that all 10 screws are proper?
(b) Assuming that all 10 screws are proper, what is the conditional probability that they come from the box with 98% proper screws?

2. An old lady knows a gossip. She tells it to 0, 1 or 2 other ladies with equal probability. Then each lady will tell it to a random number of other ladies with the same distribution independently from the others, and so on.

(a) Let $X_2$ denote the number of ladies who heard the gossip from someone who heard it from the original lady. Calculate $E(X_2)$ and $P(X_2 = 0)$.
(b) What is the probability that the gossip will keep spreading forever?
(c) Calculate the expected number of ladies who hear the gossip.

3. An average of 3 requests per second arrive to a server. On average, 1/4 of the requests is type A and 3/4 of the requests is type B.

(a) Calculate the probability that exactly 2 type A requests arrive during a 2 second interval.
(b) Assuming that 2 type A requests arrive during a 2 second interval, calculate the probability that both arrive during the first half of the interval.
(c) Assuming that 4 requests arrive in total during a 2 second interval, calculate the probability that 2 of them are type A.

4. A doctor spends on average 4 minutes with each patient, and never more than 10 minutes. Give a large deviation estimate on the probability that the time needed to attend to 50 patients is more than 5 hours.

5. At a sports match, people buy soda in the bar of the stadium during the break. The mean soda consumption of each person is 0.6 liter and the deviation is 0.4 liter. Assuming that 30000 people attend the game, what is the amount of soda the pub should prepare with for the game if they want to keep the probability of running out of soda below 3%?

Total working time is 90 minutes. Each problem is worth 9 points.