1. We have two large boxes containing candies. One box contains 20% red and 80% blue candies, the other box contains 40% red and 60% blue candies, but we don’t know which box is which. We open one of the boxes at random and take out 3 candies. Assuming that all 3 candies are blue, calculate the conditional probability that they are from the box containing 20% red candies.

2. A computer virus is spreading. Initially, there is a single infected computer. Before the virus is discovered and removed, each infected computer infects a random number of computers with distribution $\text{PGeo}(0.49)$ before the virus is discovered and removed, independent from other computers.

   (a) Model the scenario with a branching process. Is the process subcritical, critical or supercritical?
   (b) Calculate the probability that the virus will stop spreading eventually.
   (c) What is the expectation of the total number of infected computers?

3. On a road, on average 1 truck and 3 cars pass per minute (we assume no other vehicles pass). We stand by the road and count the vehicles.

   (a) Calculate the probability that during a 30 second interval, exactly 1 vehicle (either car or truck) passes.
   (b) Calculate the probability that we have to wait less than 1 minute for the next truck.
   (c) Assuming that 2 vehicles arrive in a 2 minute interval, calculate the conditional probability that exactly 1 of them arrives during the first minute and it is a car.

4. A server receives on average 1 request per second. Give an estimate on the probability that during 1 hour, the server receives more than 3700 requests.

5. In a given area, an internet service provider (ISP) has two types of clients:

   • basic users, who use a maximum bandwidth of 10 MBps, and average bandwidth 2 MBps during peak hours;
   • advanced users, who use a maximum bandwidth of 30 MBps, and average bandwidth 5 MBps during peak hours.

The ISP has 2500 basic users and 1000 advanced users. Give an upper bound on the total bandwidth that should be provided for the users in this area so that the values provided above are met with at least 99.99% probability at a given time during peak hours.
1. Let $N \sim \text{POI}(10)$ and $Y = X_1 + \cdots + X_N$ where the $X_i$'s are independent from each other and $N$, with identical distribution

$$P(X_i = 0) = P(X_i = 1) = 1/3, \quad P(X_i = 2) = P(X_i = 3) = 1/6.$$  
Calculate the probability generating function and mean of $Y$.

2. Dennis writes a computer program, but the program has an error. He tries to correct it. He succeeds with probability 0.4, but with probability 0.6, the old error remains and he also adds a new error. In each round, if the program still contains errors, he tries to correct each error, but for each error, he corrects it with probability 0.4, but with probability 0.6, he adds a new error (and the old error also remains).

(a) Calculate the probability that he finishes in at most 3 rounds (the first round is when he tries to correct the original error).

(b) What is the expected number of errors after 3 rounds?

(c) What is the probability that the program will eventually run without errors?

3. During hoeing (a type of gardening work), Joe finds on average 1 may-beetle and 2 potato-beetles per hour. There are no other bugs in Joe's garden.

(a) Calculate the probability that he finds exactly 2 beetles over 2 hours.

(b) He accidentally kills each beetle with his hoe with probability 1/2. Calculate the probability that over 2 hours, he kills exactly 2 may-beetles and every potato-beetle he finds.

(c) What is the distribution of the time when he finds the first beetle alive (not killed by the hoe)?

4. A server receives an average of 1 request per second. Give an upper bound on the probability that during 1 hour, it receives at least 5000 requests. (Help: $\text{EXP}(\mu)$ has a Cramer rate function $I(x) = \mu x - 1 - \ln(\mu x)$, and $\text{POI}(\lambda)$ has a Cramer rate function $I(x) = x \ln \frac{\lambda}{x} - x + \lambda$ (for $x > 0$).)

5. A test has 5 questions, each of them worth a maximum of 9 points. The teacher marks each question by rolling a number with a fair six-sided die and assigning the number as the score for that question. Estimate the probability that in a class of 40 students, the average score per student is at least 20.