Stochastics
Problems for midterm test 2

1. Jack works at a company. Employees of the company are put into 3 categories: good, great and fantastic (in that order; fantastic is the best category). Each month, Jack gets promoted by 1 category with probability $\frac{1}{3}$ (if he was already fantastic, he stays there), gets demoted by 1 category with probability $\frac{1}{6}$ (if he was good, he stays there) and stays in the same category with probability $\frac{1}{2}$.

(a) Model Jack’s monthly job category with a discrete time Markov chain. Calculate the transition matrix.

(b) This month, he is great. What is the probability that 2 months from now, he will be great again?

(c) In the long run, how much of the time does he spend in fantastic?

(d) Monthly salary is $300000$ HUF for good employees, $320000$ HUF for great employees and $350000$ HUF for fantastic employees. Calculate the long term average salary Jack gets.

2. Anna is taking a test and she is panicking. The test has 3 problems. She spends a random exponentially distributed time with each problem (regardless of the past), then switches to one of the other problems at random. The average time spent on a problem before switching is 1, 1.5 and 2 (in minutes) for problems 1, 2 and 3 respectively.

(a) Model Anna’s behaviour with a continuous time Markov chain. Calculate the generator and draw the graph representation.

(b) Calculate the stationary distribution.

(c) For each minute she spends on problem 1, she will get on average +0.5 points in the total score. For each minute spent on problem 2, she will get on average +0.6 points in the total score, and for each minute spent on problem 3, she will get on average +0.8 points in the total score. Calculate her average score after 90 minutes of work.

3. People use an ATM machine in downtown Budapest. On average, one person arrives every 2 minutes to use the ATM. If someone is already using the ATM, further people will wait in line unless there are at least 3 people there (including the one currently using the ATM); if a person arrives with 3 people already there, he leaves immediately and does not come back. Each person uses the ATM for 1 minute on average.

(a) Model the usage of the ATM with a continuous time Markov chain. What are the states? Calculate the generator.

(b) Calculate the stationary distribution.

(c) What is the probability that at a random time, a person is using the ATM with nobody else waiting?

(d) In the long run, what portion of the time is the ATM in use?

(e) What is the long-term average length of the queue at the ATM (including the person using it)?

4. Cars are passing by on a road with independent exponentially distributed interarrival times. The parameter $\lambda$ of the exponential distribution is unknown ($\lambda$ is measured in $1/sec$). We register the following interarrival times (in seconds): 8, 15, 4, 13, 35. Calculate the moment estimate for $\lambda$ based on the sample.

5. When someone takes an IQ test, the result is random, with expectation equal to his or her real IQ, and deviation equal to 3. Jack claims that he has an IQ of 120. He takes 5 separate IQ tests to prove it; his results are 118, 116, 119, 119, 118. Test the hypothesis that Jack has an IQ equal to 120 against the hypothesis that his IQ is not 120 with a confidence level of 95%.