1. Alice sends Bob a letter. Once Alice throws the letter in the mailbox, the postman takes the letter into the post office with probability $\frac{1}{3}$ each day (regardless of the past). After the postman takes it into the post office, the post office processes it either the next day or in 2 days (with probability $\frac{1}{2}$ each). After the letter is processed, it is delivered by another postman to Bob with probability $\frac{2}{5}$ each day (regardless of the past). Let $X$ denote the number of days between Alice sending the letter and Bob receiving the letter (the minimum value of $X$ is $0+1+0$). Calculate the probability generating function and expectation of $X$.

2. Jack has a random number of children with pessimistic geometric distribution with parameter $p = 0.4$. Each of his descendants has a number of children with the same distribution, independent from the others.

   (a) Model the scenario with a branching process. Is the process subcritical, critical or supercritical?

   (b) What is the expected number of Jack’s grandchildren?

   (c) What is the probability that Jack’s descendants die out eventually?

3. Type A and type B packets arrive at a server. On average, 1 type A packet arrives per second, and 0.5 type B packets arrive per second.

   (a) Calculate the probability of the following event: at least 2 packets arrive in a 1 second interval, and no packet arrives in the next 1 second interval.

   (b) Calculate the probability that the first two packets arriving are both of type A.

4. A teacher is marking tests. The average time he spends marking a test is 5 minutes, but he never spends more than 10 minutes on a test. The test was taken by 40 students. Give a large deviation estimate on the probability that marking all 40 tests takes more than 5 hours.

5. The amount of sugar in a pack is random, with mean 1000 and deviation 20 (measured in grammes). Estimate the probability that the average weight of 25 packs is less than 990 grammes.
Stochastics
Problem sheet D
Fall 2017

1. In a population, 1 out of 10000 people is infected with a disease. A test for the
disease gives a false result with probability 1% (so if the tested person is healthy,
the result will be negative with 99% probability, and if the tested person is infected,
the result will be negative with 1% probability). Assuming Dennis tests positive,
what is the probability that he has the disease?

2. A reviewer is given a text that contains a random number of errors with distribution
POI(5). He finds each error with probability 0.9, independent from the others.
(a) What is the distribution of the number of errors he finds?
(b) Calculate the probability that he finds all errors in the text.

3. We model the spreading of cancer. Initially, there is a single cancerous cell. Before
it dies, it spreads to a random number of cells with distribution pessimistic geo-
metric with parameter 0.52. Then each cancerous cell displays the same behaviour,
independent from the others.
(a) Model the scenario with a branching process. Is the process subcritical, critical
or supercritical?
(b) Calculate the probability that the cancerous cells stop spreading eventually.
(c) What is the expectation of the total number of cancerous cells?

4. A server receives on average 1 request per second. Give an estimate on the proba-
bility that during 1 hour, the server receives more than 3700 requests.

5. We keep rolling a regular six-sided die until we get 1000 sixes. Give a large de-
viation estimate on the probability that this happens in 5000 rolls or less. (Help:
Bernoulli(p) has Cramér rate function \( I(x) = x \ln((x(1 - p)/(p(1 - x))) + \ln((1 - x)/(1 - p))) \), and GEO(p) has Cramér rate function \( I(x) = x \ln((x - 1)/(x(1 - p))) + \ln((1 - p)/(p(x - 1))) \).)