Stochastics
Sample problem sheet for midterm test 1 A
Fall 2021

1. We have two coins. One of them is a regular coin, with tails on one side and heads on the other side, but the other coin has heads on both sides. Both coins are fair (fall on each side with 50% probability when flipped).
    (a) We pick one of the coins at random and flip it 3 times. What is the probability that the result will be 3 heads?
    (b) We pick one of the coins at random and flip it 3 times. Assuming the result is heads, what is the conditional probability that it was the regular coin?
2. Any computer infected with a certain computer virus will infect a random number of further computers with distribution PGEO(1/3) before the virus is found and removed. Initially, the virus is present only on a single computer. We call the computers infected by that computer generation 1; generation 2 contains the computers infected by computers in generation 1, and so on. X_k denotes the number of computers in generation k.
    (a) Model the situation with a branching process. What are the individuals? Is the process subcritical, critical or supercritical?
    (b) What is the expectation of X_4?
    (c) What is the probability that the virus stops spreading eventually?
3. On average, 2.5 cars per minute arrive to a parking lot.
    (a) Calculate the probability that exactly 4 cars arrive over a 2 minute interval.
    (b) What is the conditional probability that within a 2 minute interval, 2 cars arrive during the first minute of the interval, assuming 4 cars arrive in total during the 2 minute interval?
4. 400 guests are at a restaurant that serves 2 types of menu, A and B. The restaurant has enough stock to prepare 280 portions of menu A and 500 portions of menu B. Each guest will choose menu A with probability 60% and menu B with probability 40%, independently from the other guests. Estimate the probability that the restaurant can serve every guest their choice of menu.
5. In a given area, an internet service provider (ISP) has two types of clients:
    - basic users, who use a maximum bandwidth of 10 MBps, and average bandwidth 2 MBps during peak hours;
    - advanced users, who use a maximum bandwidth of 30 MBps, and average bandwidth 5 MBps during peak hours.

    Assuming the ISP has 3000 basic users and 1000 advanced users, give an upper bound on the total bandwidth that should be provided for the users in this area so that the values provided above are met with at least 99.99% probability at a given time during peak hours.

Each problem is worth 9 points.

Stochastics
Sample problem sheet for midterm test 1 B
Fall 2021

1. A voter poll is taken in two states to measure the support for two parties: A and B. In state 1, 60% of voters support party A and 40% of voters support party B. In state 2, 70% of the voters support party A and 30% support party B. From among the total population of the two states, 30% live in state 1 and 70% live in state 2.

   (a) What is the probability that a random voter from among the two states supports party A?

   (b) Assuming that a random voter from among the two states supports party A, what is the conditional probability that the voter lives in state 1?

2. Sam will have 0 children with probability 0.2, 1 child with probability 0.4, and 2 children with probability 0.4. Then his children will each have a random number of children with the same distribution, independent of others.

   (a) Let X_2 denote the number of Sam's grandchildren. Calculate $\mathbf{E}(X_2)$ and $\mathbf{P}(X_2 = 0)$.

   (b) What is the probability that Sam's family tree eventually dies out?

   (c) Calculate the expected number of Sam's descendants.

3. In a video game, item drops can be either weapons or gold. On average, there are 2 weapon drops per hour and 6 gold drops per hour.

   (a) Calculate the probability that during 20 minutes of gameplay, exactly 3 items drop (of any type).

   (b) Assuming that during 20 minutes of gameplay, exactly 3 items drop (of any type), what is the conditional probability that all 3 items are gold?

   (c) What is the probability that during 20 minutes of gameplay, all item drops are weapons?

4. A doctor spends on average 5 minutes attending to a patient, but never more than 15 minutes. Give a large deviation bound on the probability that attending to 80 patients takes more than 10 hours.

5. A student takes a test with 100 questions. For each question, the student answers correctly with probability 0.8, independent from the other questions. Estimate the probability that the student answers at least 85 questions correctly.

Each problem is worth 9 points.