2nd make-up midterm
2021.12.10.

1. W_o := weight of orange 3 bag $\quad W_o \sim N(2, 0{,}2)$ in kg
W_a := weight of apple 3 bag $\quad W_a \sim N(2, 0{,}2)$
} they are independent

P_o := price of the bag of oranges

P_a := price of the bag of apples in Ft

$\mathbb{P}$(the total cost is less than 2000 Ft) =

$= \mathbb{P}(P_o + P_a < 2000) =$ 1p

2p

$= \mathbb{P}(500 \cdot W_o + 600 \cdot W_a < 2000) \overset{\text{standardize}}{=} \mathbb{P}\left(\frac{500W_o + 600W_a - 2200}{\sqrt{0{,}2 \cdot (500^2 + 600^2)}} < \frac{-200}{\sqrt{0{,}2 \cdot (500^2+600^2)}}\right) =$ 2p

$500 \cdot W_o \sim N(500 \cdot 2, 500^2 \cdot 0{,}2)$ $\quad 600 \cdot W_a \sim N(600 \cdot 2, 600^2 \cdot 0{,}2)$

$\frac{500W_o + 600W_a - 2200}{\sqrt{0{,}2 \cdot (500^2 + 600^2)}} \sim N(0,1)$ since W_o and W_a are independent

$\frac{-200}{\sqrt{0{,}2 \cdot (500^2+600^2)}} \approx -0{,}57$

$= \Phi(-0{,}57) = 1 - \Phi(0{,}57) = 0{,}2843$ 2p

$\Phi(0{,}57) \approx 0{,}7157$

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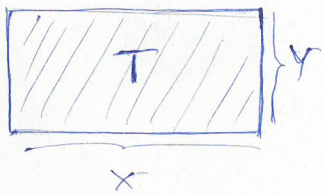
2. $X \sim \text{Uni}[0,3]$, $Y \sim \text{Uni}[0,3]$ independent random variables.

Obviously, $0 \le T \le 9$. Let $t \in [0,9]$.



$F_T(t) = \mathbb{P}(T < t) = \mathbb{P}(XY < t) =$ 3p

$= \int_0^{t/3}\int_0^3 \underbrace{\frac{1}{3}\cdot\frac{1}{3}}_{\frac{1}{9}} dy\,dx + \int_{t/3}^3 \int_0^{t/x} \frac{1}{3}\cdot\frac{1}{3}\,dy\,dx =$

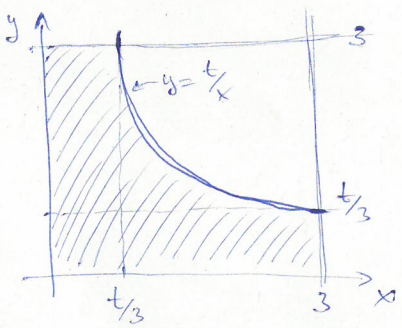
the joint density function of X, Y.

$= \frac{1}{9}\left(\int_0^{t/3} 3\,dx + \int_{t/3}^3 \frac{t}{x}\,dx\right) = \frac{1}{9}\cdot 3 \cdot \frac{t}{3} + \frac{1}{9}t \cdot [\ln x]_{x=t/3}^{x=3} =$ 3p

$= \frac{t}{9}\left(1 + \ln 3 - \ln\frac{t}{3}\right) = \frac{t}{9}(1 + \ln 9 - \ln t)$

$= -\ln t + \ln 3$

The set of possible events where $XY < t$:



3. 7 mathematicians
11 physicians

$i \in \{1, \dots, 7\}$: $X_i := \begin{cases} 1, & \text{if the } i^{th} \text{ mathematician only has mathematician teammates} \\ 0, & \text{otherwise} \end{cases}$ 1p

$X :=$ the number of those mathematicians who only have mathematician teammates

$$\underline{\underline{\mathbb{E}[X]}} = \mathbb{E}\Big[\sum_{i=1}^{7} X_i\Big] = \sum_{i=1}^{7} \mathbb{E}[X_i] = \sum_{i=1}^{7} \underbrace{\mathbb{P}(X_i = 1)}_{= \binom{6}{2} / \binom{17}{2}} = 7 \cdot \frac{6}{17} \cdot \frac{5}{16} \approx \underline{\underline{0{,}77}}$$

1p 2p 3p

"good cases": the 2 teammates are chosen from the other 6 mathematicians

"all cases": the teammates are chosen from the other 17 students

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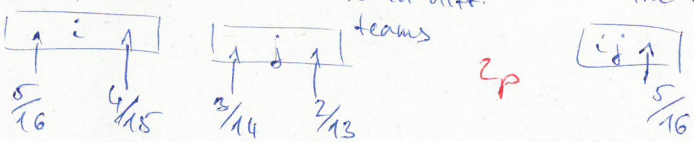
Bonus/ $\mathrm{Var}(X) = \mathbb{E}[X^2] - \underbrace{\mathbb{E}^2[X]}_{= 0{,}77^2}$

$$\mathbb{E}[X^2] = \mathbb{E}\Big[\Big(\sum_{i=1}^{7} X_i\Big)^2\Big] = \sum_{i=1}^{7} \underbrace{\mathbb{E}[X_i^2]}_{= \mathbb{E}[X_i]} + 2 \sum_{i<j} \underbrace{\mathbb{E}[X_i X_j]}_{= \mathbb{P}(X_i X_j = 1)} =$$

i and j are in diff. teams

i and j are in the same team



2p

$$= 0{,}77 + 2 \cdot \binom{7}{2} \underbrace{\Big(0{,}0027 + \frac{5}{16}\Big)}_{\approx 0{,}32} \approx 14{,}21 \implies \underline{\underline{\mathrm{Var}(X)}} = 14{,}21 - 0{,}77^2 \approx \underline{\underline{13{,}62}}$$

2p