1 3. HF megoldasa:

1/(b): $f(-x) = |\sin(-x)| = |-\sin(x)| = |\sin(x)| = f(x)$, tehat f paros fuggveny, igy $b_k = 0$, ha $k = 1, 2, \ldots$.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} |\sin(x)|\; dx = \frac{1}{\pi}\int_0^{\pi} \sin(x)\, dx = \frac{1}{\pi}\left[-\cos(x)\right]_0^{\pi} = \frac{1}{\pi}(-(-1)-(-1)) = \frac{2}{\pi}$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} |\sin(x)|\cos(kx)\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin(x)|\cos(kx)\, dx = \frac{2}{\pi}\int_0^{\pi} \sin(x)\cos(kx)\, dx =$$
$$\frac{2}{\pi}\int_0^{\pi} \frac{1}{2}\left(\sin((1+k)x) + \sin((1-k)x)\right)\, dx = \frac{1}{\pi}\int_0^{\pi} \sin((k+1)x)\, dx - \frac{1}{\pi}\int_0^{\pi} \sin((k-1)x)\, dx$$

Tudjuk, hogy

$$\int_0^{\pi} \sin(mx)\, dx = \begin{cases} \frac{1-(-1)^m}{m} & \text{ha } m = 1, 2, 3, \ldots \\ 0 & \text{ha } m = 0 \end{cases} \tag{1}$$

Ezt felhasznalva kapjuk, hogy

$$a_1 = \frac{1}{\pi}\int_0^{\pi} \sin(2x)\, dx - \frac{1}{\pi}\int_0^{\pi} \sin(0x)\, dx = 0 - 0 = 0$$

Ha pedig $k \geq 2$, akkor

$$a_k = \frac{1}{\pi}\int_0^{\pi} \sin((k+1)x)\, dx - \frac{1}{\pi}\int_0^{\pi} \sin((k-1)x)\, dx =$$
$$\frac{1}{\pi}\left(\frac{1-(-1)^{k+1}}{k+1} - \frac{1-(-1)^{k-1}}{k-1}\right) = \begin{cases} 0 & \text{ha } k \text{ paratlan} \\ \frac{-4}{\pi}\frac{1}{k^2-1} & \text{ha } k \text{ paros} \end{cases}$$

Tehat

$$a_0 = \frac{2}{\pi} \quad a_1 = 0 \quad a_2 = \frac{1}{\pi}\frac{-4}{3} \quad a_3 = 0 \quad a_4 = \frac{1}{\pi}\frac{-4}{15}$$

3/(b)

$$f(x) = \sin^2(x) + \sin^3(x) = \frac{1-\cos(2x)}{2} + \sin(x)\frac{1-\cos(2x)}{2} =$$
$$\frac{1}{2} - \frac{1}{2}\cos(2x) + \frac{1}{2}\sin(x) - \frac{1}{2}\sin(x)\cos(2x) =$$
$$\frac{1}{2} - \frac{1}{2}\cos(2x) + \frac{1}{2}\sin(x) - \frac{1}{2}\left(\sin(3x) - \sin(x)\right) =$$
$$\frac{1}{2} + \frac{3}{4}\sin(x) - \frac{1}{2}\cos(2x) - \frac{1}{4}\sin(3x)$$

Tehat

$$a_0 = \frac{1}{2} \quad a_1 = 0 \quad a_2 = -\frac{1}{2} \quad a_3 = a_4 = a_5 = \cdots = 0$$
$$b_1 = \frac{3}{4} \quad b_2 = 0 \quad b_3 = -\frac{1}{4} \quad b_4 = b_5 = b_6 = \cdots = 0$$