1 a) $dY_t = 2Y_t dB_t + 4Y_t dt = Y_t(2 dB_t + 4dt) \qquad Y_0 = 3$

The differential form of the SDE is in $dY_t = Y_t dX_t$

so the solution is $Y_t = 3\exp(X_t - X_0 - \frac{1}{2}[X]_t) =$

$$= 3\exp\Big(\underbrace{\int_0^t 4\,ds + \int_0^t 2\,dB_s}_{X_t - X_0} - \underbrace{\frac{1}{2}\int_0^t 2^2\,ds}_{\frac{1}{2}[X]_t}\Big) = 3\exp(2t + 2B_t)$$

b) Want: the dt term vanishes in $dM_t = d(f(t)Y_t)$

$$d(f(t)Y_t) = f'(t)\,dt\,Y_t + f(t)\,dY_t = f'(t)Y_t\,dt + f(t)(Y_t(2dB_t + 4$$

$$= (f'(t) + 4f(t))Y_t\,dt + 2f(t)Y_t\,dB_t$$

$f'(t) + 4f(t) = 0$

$f'(t) = -4f(t)$

$f(t) = c\,e^{-4t}$

2: a) $Z_t = \int_0^t (t-u)(t+u)\, dB_u = \int_0^t t^2 - u^2 \, dB_u = t^2 B_t - \int_0^t u^2 \, dB_u$

$dZ_t = 2t\, dt\, B_t + t^2 dB_t - t^2 dB_t = 2t B_t \, dt$

$Z_t = 0 + \int_0^t 2u B_u \, du + \int_0^t 0 \, dB_u = \int_0^t 2u B_u \, du$

b) $[Z]_t = \int_0^t 0^2 \, ds = 0$

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c) $Var(Z_2) = \mathbb{E}\left(\left(\int_0^2 4 - u^2 \, dB_u\right)^2\right) \overset{\downarrow}{=} \mathbb{E}\left(\int_0^2 (4-u^2)^2 \, du\right) =$

$= \int_0^2 16 - 8u^2 + u^4 \, du = \left[16u - \frac{8}{3}u^3 + \frac{u^5}{5}\right]_0^2 = 32 - \frac{64}{3} + \frac{32}{5}$