Midterm Exam - October 11, 2018, Stochastic Analysis, GROUP A

Family name ________________________ **Given name** ________________________

Signature ________________________ **Neptun Code** ________________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. Let $Y_1, Y_2, \ldots$ denote i.i.d. random variables with distribution

$$\mathbb{P}(Y_k = 1) = \frac{3}{4}, \qquad \mathbb{P}(Y_k = -1) = \frac{1}{4}.$$

Let $X_n = 1 + Y_1 + \cdots + Y_n$. Let $\mathcal{F}_n = \sigma(Y_1, \ldots, Y_n)$. For $x \in \mathbb{Z}$, Let $T_x := \inf\{\, n \,:\, X_n = x \,\}$.

(a) (2 marks) Find a constant $C \neq 1$ such that $\left(C^{X_n}\right)$ is a martingale.

(b) (2 marks) Give a formula for $q := \mathbb{P}(T_0 < T_6)$ using the optional stopping theorem.
Instruction: You don't have to verify that the optional stopping theorem can be applied here.
Instruction: It is enough to give a correct formula which contains C as a parameter.

(c) (2 marks) Find a constant m such that $(X_n - mn)$ is a martingale.

(d) (2 marks) Give a formula for $\mathbb{E}(\min\{T_0, T_6\})$ using the optional stopping theorem.
Instruction: You don't have to verify that the optional stopping theorem can be applied here.
Instruction: It is enough to give a correct formula which contains q and m as parameters.

2. Let $(B(t))$ denote a standard Brownian motion.
Recall the definition of the stationary O.-U. process $X_t = e^{-2t} B(e^{4t})$.

(a) (2 marks) Find the covariance matrix of (X_{-1}, X_1).

(b) (2 marks) Calculate the conditional expectation of X_1 given the σ-algebra generated by X_{-1}.

(c) (3 marks) Calculate the conditional expectation of X_{-1} given the σ-algebra generated by X_1.