Midterm Exam - October 11, 2018, Stochastic Analysis, GROUP B

Family name ________________________ **Given name** ________________________

Signature ________________________ **Neptun Code** ________________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. Let $(B(t))$ denote a standard Brownian motion.
   Recall the definition of the stationary O.-U. process $Y_t = e^{-t}B(e^{2t})$.
   (a) (2 marks) Find the covariance matrix of (Y_{-3}, Y_3).
   (b) (2 marks) Calculate the conditional expectation of Y_3 given the σ-algebra generated by Y_{-3}.
   (c) (3 marks) Calculate the conditional expectation of Y_{-3} given the σ-algebra generated by Y_3.

2. Let $X_1, X_2, \dots$ denote i.i.d. random variables with distribution

$$\mathbb{P}(X_k = 1) = \frac{4}{5}, \qquad \mathbb{P}(X_k = -1) = \frac{1}{5}.$$

   Let $Z_n = 1 + X_1 + \cdots + X_n$. Let $\mathcal{F}_n = \sigma(X_1, \dots, X_n)$. For $x \in \mathbb{Z}$, Let $T_x := \inf\{\, n \;:\; Z_n = x \,\}$.
   (a) (2 marks) Find a constant $\alpha \neq 1$ such that $\left(\alpha^{Z_n}\right)$ is a martingale.
   (b) (2 marks) Give a formula for $p := \mathbb{P}(T_0 < T_7)$ using the optional stopping theorem.
   Instruction: You don't have to verify that the optional stopping theorem can be applied here.
   Instruction: It is enough to give a correct formula which contains α as a parameter.
   (c) (2 marks) Find a constant μ such that $(Z_n - \mu n)$ is a martingale.
   (d) (2 marks) Give a formula for $\mathbb{E}(\min\{T_0, T_7\})$ using the optional stopping theorem.
   Instruction: You don't have to verify that the optional stopping theorem can be applied here.
   Instruction: It is enough to give a correct formula which contains p and μ as parameters.