Midterm Exam - November 22, 2018, Stochastic Analysis,   GROUP A

Family name ______________________ **Given name** ______________________

Signature ______________________ **Neptun Code** ______________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. (a) (5 marks) Find the covariance of $\int_0^3 (2 - 7B_u)\,\mathrm{d}B_u$ and $\int_0^5 (2B_u^2 - 1)\,\mathrm{d}B_u$.

   (b) (2 marks) Find a simple closed formula for the value of $\int_0^3 (2 - 7B_u)\,\mathrm{d}B_u$.

2. Let us define

$$X_t = \frac{1}{\sqrt{4-t}} \exp\left(\frac{B_t^2}{2t-8}\right), \qquad 0 \le t \le 2.$$

   (a) (4 marks) Show that $(X_t)_{0 \le t \le 2}$ is a martingale.

   (b) (4 marks) Let $\mathcal{F}_t = \sigma\,(B_s,\, 0 \le s \le t)$ denote the sigma-algebra generated by the Brownian motion up to time t. Find the constant C and the process $(Y_t)_{0 \le t \le 2}$ adapted to the filtration $(\mathcal{F}_t)_{0 \le t \le 2}$ such that

$$\exp\left(-\frac{1}{4}B_2^2\right) = C + \int_0^2 Y_s\,\mathrm{d}B_s.$$