Midterm Exam (second midterm) - December 6, 2018, Stochastic Analysis

Family name _____ **Given name** _____

Signature _____ **Neptun Code** _____

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. (a) (4 marks) Find a simple explicit formula (in terms of t and B_t) for the non-negative process (Y_t) satisfying
$$Y_t = 3 + 2\int_0^t Y_s \,\mathrm{d}B_s + 4\int_0^t Y_s \,\mathrm{d}s$$
Hint: First calculate $\mathrm{d}\ln(Y_t)$.

   (b) (4 marks) Find a deterministic function $f : \mathbb{R}_+ \to \mathbb{R}$ with $f(0) = 1$ such that (M_t) is a martinage, where
$$M_t = f(t)Y_t$$

2. Let us define $Z_t = \int_0^t (t-u)(t+u)\mathrm{d}B_u$ for any $t \geq 0$.

   (a) (3 marks) Show that (Z_t) is an *Itô process* by rewriting it in the form $Z_t = Z_0 + \int_0^t \mu_u \mathrm{d}u + \int_0^t \sigma_u \mathrm{d}B_u$.

   (b) (2 marks) Calculate the quadratic variation $[Z]_t$.

   (c) (2 marks) Calculate the variance of Z_2.